RECURRING DECIMALS

<u>Aims:</u>
This starter activity is all about the discussion following a simple calculator 'trial and improvement' exercise.

<u>Equipment:</u> Calculators.

We all know that the infinitely recurring decimal 0.333333… (written $0.\dot{3}$) is the same as $\frac{1}{3}$ (or simply 1 ÷ 3).

Similarly, 0.666666… = $\frac{2}{3}$ or 2 ÷ 3.

(Consider using long division to confirm the above facts etc.)

Use a calculator (and some trial and error!) to convert the following into fractions:

i) 0.111111… ii) 0.444444… iii) 0.555555…

iv) 0.999999… v) 0.121212… vi) 0.353535…

<u>Discussion.</u>

i) In linking the values of 0.111111… and 0.333333…, it is possible to quickly see that 0.111111… = $\frac{1}{9}$ without any trial and error at all!

ii) 0.444444… = 4 × 0.111111… and thus 0.444444… = $\frac{4}{9}$.

iii) 0.555555… = 5 × 0.111111… and thus 0.555555… = $\frac{5}{9}$.

<u>Alternatively:</u> 0.555555… = 0.111111… + 0.444444… = $\frac{1}{9} + \frac{4}{9}$ etc.

iii) 0.999999… = 9 × 0.111111… = $\frac{9}{9}$ = 1 !!!

Why? How? What the…? **This is a good one to discuss!**

iv) For many, some serious guesswork is required here.
A hint would be to ask the class to first consider 0.010101… etc.

Answer = $\frac{12}{99}$ or $\frac{4}{33}$.

v) Answer = $\frac{35}{99}$.

Extension.

Use algebra (as required for higher GCSE).

E.g. i) Put $x = 0.111111\ldots$, then $10x = 1.111111\ldots = 1 + x$.
Solve to get $9x = 1$
and thus $x = \frac{1}{9}$.

v) Put $x = 0.121212\ldots$, then $100x = 12.121212\ldots = 12 + x$.
Solve to get $99x = 12$
and thus $x = \frac{12}{99}$ etc.

etc.

Give a few questions and then ask the class to try and adapt the method to convert the following to fractions:

i) 0.5222222… ii) 0.3111111… iii) 0.1999999…
etc.

Offer a prize for those able to adapt the algebra accordingly!