

## HYPOTHESIS TESTING practice sheet

- 1) The scores obtained in a particular test taken by a random sample of 49 students can be summarised by  $\sum x = 3356.5$  and  $\sum x^2 = 231648.25$ .
- Calculate unbiased estimates for the mean  $\mu$  and the variance  $\sigma^2$  of the test scores.
  - In previous years, the mean score obtained in the same test was 70. Is there sufficient evidence, at the 5% level, to suggest that these latest students have not performed as well as expected?
- 2) The scores obtained in a particular test taken by a random sample of 30 students can be summarised by  $\sum x = 1740$  and  $\sum x^2 = 102776$ .
- Calculate unbiased estimates for the mean  $\mu$  and the variance  $\sigma^2$  of the test scores.
  - In previous years, the mean score obtained in the same test was 55. Is there sufficient evidence, at the 5% level, to suggest that these latest students have performed better than expected?
- 3) 'Stay awake' coffee is packed in jars which are supposed to have an average weight of 900g.
- A random sample of 60 jars have masses which can be summarised by  $\sum x = 53868$  and  $\sum x^2 = 48368590$ .
- Test, at the 5% level, whether the coffee jars are being filled correctly and precisely.
- 4) 'Happy Hippie' coffee is packed in jars which are supposed to have an average weight of 1500g.
- A random sample of 40 jars have masses which can be summarised by  $\sum x = 59600$  and  $\sum x^2 = 88862500$ .
- A disgruntled customer complains that the jars of coffee are, on average, underweight.
- Test, at the 5% level, whether the customer is correct.
- 5) A shopkeeper complains that the average weight of chocolate bars of a certain type that he is buying from a wholesaler is less than the stated value of 8.50 g. The shopkeeper weighed 100 bars from a large delivery and found that their weights had a mean of 8.36 g and a standard deviation of 0.72 g. Using a 5% significance level, determine whether or not the shopkeeper is justified in his complaint.
- State clearly the null and alternative hypotheses that you are using.
- Calculate the smallest level of test that would result in your null hypothesis being rejected.
- {Hint: the standard deviation of 0.72 is the value of  $s$ , not  $\sigma$ . You will need to calculate the unbiased estimator for  $\sigma^2$  yourself.}**

## ANSWERS.

- 1) i)  $\hat{\mu} = 68.5$ ,  $\hat{\sigma}^2 = 36$ . ii)  $\{Z_{\text{test}} = -1.75$ , critical region  $Z < -1.645\}$  Reject  $H_0$ . There is sufficient evidence at the 5% level to suggest that these students have not performed as well as expected.
- 2) i)  $\hat{\mu} = 58$ ,  $\hat{\sigma}^2 = 64$ . ii)  $\{Z_{\text{test}} = 2.054$ , critical region  $Z > 1.645\}$  Reject  $H_0$ . There is sufficient evidence at the 5% level to suggest that these students have performed better than expected.
- 3)  $\hat{\mu} = 897.8$ ,  $\hat{\sigma}^2 = 99.99322034$ .  $\{Z_{\text{test}} = -1.704$ , critical region  $Z < -1.96$  or  $Z > 1.96\}$   
Accept  $H_0$ . There is insufficient evidence at the 5% level to suggest that the coffee jars are not being filled correctly
- 4)  $\hat{\mu} = 1490$ ,  $\hat{\sigma}^2 = 1500$ .  $\{Z_{\text{test}} = -1.633$ , critical region  $Z < -1.645\}$   
Accept  $H_0$ . There is insufficient evidence at the 5% level to suggest that the coffee jars are being under filled.
- 5)  $H_0: \mu = 8.50$ ,  $H_1: \mu < 8.50$ .  $\hat{\sigma}^2 = 0.52363636$ .  $\{Z_{\text{test}} = -1.935$ , critical region  $Z < -1.645\}$   
Reject  $H_0$ . There is sufficient evidence at the 5% level to support the shopkeepers claim.  
Required level = 2.65%.