TYPE I AND TYPE II ERRORS

1) The random variable X has a normal distribution with variance 36. Ten independent measurements of X are made in order to test the null hypothesis $\mu = 100$ against the alternative hypothesis $\mu > 100$, where μ is the mean of X.

Find the probability of making a Type II error when applying a significance test at the 5% level if the true mean is 105.

2) The random variable X has a normal distribution with variance 64. Twenty independent observations of X are made in order to test the null hypothesis $\mu = 25$ against the alternative hypothesis $\mu > 25$, where μ is the mean of X.

Find the probability of making a Type II error when applying a significance test at the 5% level if the true mean is 30.

3) 'Crimson flower' jam comes in either cherry or raspberry flavours and are packed in identical sized jars in such a way that the mean mass of a cherry jar is 0·5kg and the mean mass of a raspberry jar is 0·7kg. The masses of both types of jar has standard deviation 0·5kg.

i) A box of 25 jars has mean weight 0·67kg but it is not clear which type of jam they contain. Use a 5% level of significance to test the following hypotheses:

H_0: the jars contain cherry jam,
H_1: the jars contain raspberry jam. **State your conclusion clearly.**
{Hint: one–tailed!}

ii) Calculate the probability of making a type II error.

4) A single observation x is made from a normal distribution with unknown mean μ and variance 24 to test the null hypothesis $\mu = 25$ against the alternative hypothesis that $\mu \neq 25$.
The critical region is taken to be $\{x < 21 \text{ or } x > 29\}$.

i) Find the probability that a Type I error occurs.

ii) Find the probability that a Type II error occurs if in fact $\mu = 21$.

5) In order to test whether a particular coin is fair, it is tossed 100 times and the number of heads obtained, X, counted. It is decided to accept the null hypothesis "H_0: the coin is fair" if no more than 58 heads are obtained..

Use the Normal approximation to the Binomial distribution to find:

i) the probability of a Type I error occurring,

ii) the probability of a Type II error occurring if in fact the probability of obtaining a head is 0·6.

6) A man claims that he can throw a six with a fair die five times out of six on the average. To test the claim he is asked to throw a fair die 120 times and his claim will be accepted if he manages to throw at least 90 sixes.

Using the hypotheses, H_0: P(man throws a six) = $\frac{5}{6}$

H_1: P(man throws a six) = $\frac{1}{6}$,

and a normal approximation to the binomial distribution, find the probability of a Type I error occurring.

7) In order to test whether a particular coin is fair, it is tossed 100 times and the number of heads obtained, X, counted. It is decided to accept the null hypothesis "H_0: the coin is fair" if more than 35 heads are obtained.

Use the Normal approximation to the Binomial distribution to find:

i) the probability of a Type I error occurring,

ii) the probability of a Type II error occurring if in fact the probability of obtaining a head is 0·3.

ANSWERS.

1) {H_0 is accepted when $\overline{x}$ < 103·12117.} P(Type II error) = 0·1611.

2) {H_0 is accepted when $\overline{x}$ < 27·94266546.} P(Type II error) = 0·1251.

3) i) Z_{test} = 1·7, reject H_0 etc. ii) {H_0 is accepted when $\overline{x}$ < 0·6645.} P(Type II error) = 0·3613.

4) i) P(Type I error) = o.4148, ii) P(Type II error) = 0·4487.

5) i) P(Type I error) = 0·0446, ii) P(Type II error) = 0·3799.

6) i) P(Type I error) = 0·0051.

7) i) P(Type I error) = 0·0019, ii) P(Type II error) = 0·1151.