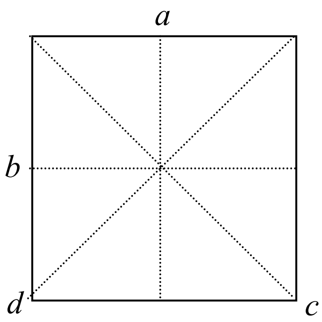
SUBGROUPS \ CYCLIC GROUPS

1) Write down the Cayley table for the group $Z_6 = \{0, 1, 2, 3, 4, 5\}$ together with the operation of addition modulo 6.

Find all of the subgroups of $\langle Z_6, +_6 \rangle$.

2) Construct the Cayley table for D_4, the symmetry group of the square using the following notation.



e = the identity element
a, b, c and d denote reflections in the lines shown.

r = "clockwise rotation of 90°"
s = "anti-clockwise rotation of 90°"
t = "rotation of 180°"

Find all the subgroups of D_4. **{There are 10 including the 2 trivial subgroups!}**

3) Show that all of the subgroups of the cyclic group $\langle Z_6, +_6 \rangle$ are themselves cyclic. {See question 1.}

4) Write down the Cayley table for the cyclic group $Z_8 = \{0, 1, 2, 3, 4, 5, 6, 7\}$ together with the operation of addition modulo 8.

Find all of the subgroups of $\langle Z_8, +_8 \rangle$ and verify that they are all cyclic.

{FACT:- the subgroups of any cyclic group are themselves cyclic!}

5) a) Write down the order of each element of D_3, the symmetry group of an equilateral triangle. Deduce that D_3 is not cyclic.

b) Verify that all of the **proper** subgroups of D_3 are cyclic.
{This shows that the converse of the above fact does not hold!}

6) A group table for the set $\{e, a, b, c\}$ is shown.

Give a simple reason why this group is not cyclic.

	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

7) The group table for the set $\{e, a, b, c\}$ is shown.

Show that this group is cyclic.

	e	a	b	c
e	e	a	b	c
a	a	b	c	e
b	b	c	e	a
c	c	e	a	b

8) Find a **<u>proper</u>** subgroup of D_4 which is not cyclic.

9) A group G has order 12.

Can there exist an element of G which has order 5? Give a reason for your answer.

10) Write down the Cayley table for the group $Z_5 = \{0, 1, 2, 3, 4\}$ together with the operation of addition modulo 5.

Find the orders of the elements 1, 2, 3 and 4 of Z_5. **Can you explain your results?**

11) G is a group of order n where is a **<u>prime</u>** number greater than 1.

i) Give a reason why G can not have any **<u>proper</u>** subgroups.

ii) Prove that G must be a cyclic group.

{**Hint:** let x be an element which is not the identity. Now look back at question 10 !}

ANSWERS.

1)

	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

Subgroups: $\{0\}$, $\{0, 3\}$, $\{0, 2, 4\}$, $\{0, 1, 2, 3, 4, 5\}$.

2)

	e	r	s	t	a	b	c	d
e	e	r	s	t	a	b	c	d
r	r	t	e	s	d	c	a	b
s	s	e	t	r	c	d	b	a
t	t	s	r	e	b	a	d	c
a	a	c	d	b	e	t	r	s
b	b	d	c	a	t	e	s	r
c	c	b	a	d	s	r	e	t
d	d	a	b	c	r	s	t	e

Subgroups: $\{e\}$, $\{e, t\}$, $\{e, a\}$, $\{e, b\}$, $\{e, c\}$, $\{e, d\}$, $\{e, r, s, t\}$, $\{e, t, a, b\}$, $\{e, t, c, d\}$
$\{e, r, s, t, a, b, c, d\}$.

3) $\{0\}$ is generated by 0. $\{0, 3\}$ is generated by 3. $\{0, 2, 4\}$ is generated by 2.
$\{0, 1, 2, 3, 4, 5\}$ is generated by 1.

4)

	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7	0
2	2	3	4	5	6	7	0	1
3	3	4	5	6	7	0	1	2
4	4	5	6	7	0	1	2	3
5	5	6	7	0	1	2	3	4
6	6	7	0	1	2	3	4	5
7	7	0	1	2	3	4	5	6

Subgroups: $\{0\}$, $\{0, 4\}$, $\{0, 2, 4, 6\}$, $\{0, 1, 2, 3, 4, 5, 6, 7\}$.

5) a) e has order 1, r has order 3, s has order 3, a has order 2, b has order 2, c has order 2.

6) All elements have order 2. No element has order 4.

7) Order of a is 4 and thus the group is cyclic.

8) $\{e, t, a, b\}$ or $\{e, t, c, d\}$ are not cyclic.

9) No. Lagrange's theorem states that the order of a subgroup must divide the order of the group.

10)

	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

All of these elements have order 5. (Lagrange's theorem!)

11) i) The order of any subgroup must be a factor of n etc.