CONTINUOUS RANDOM VARIABLES

1) The continuous random variable X has probability density function $f(x)$ where $f(x) = kx^2$ for $0 \le x \le 2$.

a) Find the value of the constant k.

b) Sketch $y = f(x)$.

c) Find $P(X \ge 1)$.

d) Find $P(0.5 \le X \le 1.5)$.

2) The continuous random variable X has p.d.f. $f(x)$ where $f(x) = k$ for $-1 \le x \le 4$.

a) Sketch $y = f(x)$.

b) Find the value of the constant k.

c) Find $P(0.5 \le X \le 2.5)$.

3) The continuous random variable X has probability density function $f(x)$ where

$$f(x) = \begin{cases} c\left(1 - \frac{1}{2}x\right) & \text{for } 0 \le x \le 2, \\ 0 & \text{otherwise.} \end{cases}$$

a) Find the value of the constant c.

b) Find $P(X \ge 1)$.

4) The continuous random variable X has p.d.f. $f(x)$ where $f(x) = k(x + 2)^2$ for $0 \le x \le 2$.

a) Find the value of the constant k.

b) Sketch $y = f(x)$.

c) Find $P(X \le 1)$ and hence find $P(X > 1)$.

5) The life, X, of a certain brand of lightbulb is modelled by the probability density function

$$f(x) = \begin{cases} ke^{-2x} & \text{for } x \ge 0, \\ 0 & \text{otherwise,} \end{cases}$$

where X is measured in thousands of hours.

a) Find the value of the constant k.

b) Find the probability that a randomly chosen lightbulb lasts longer than 1000 hours.

c) Find the probability that a randomly chosen lightbulb lasts less than 500 hours.

{**Hint:** we require $P\left(X \le \frac{1}{2}\right)$ which, in this case, equals $\int_0^{\frac{1}{2}} ke^{-2x}\,dx$ and NOT $\int_{-\infty}^{\frac{1}{2}} ke^{-2x}\,dx$ etc.

Ask, if you do not understand this!}

6) A computer ink cartridge has a life of X hours. The variable X is modelled by the probability density function

$$f(x) = \begin{cases} kx^{-2} & \text{for } x \ge 400, \\ 0 & \text{otherwise.} \end{cases}$$

a) Find the value of the constant k.

b) Find the probability that a randomly chosen cartridge has a life of at least 500 hours.

c) Find the probability that a randomly chosen cartridge will have to be replaced before 600 hours of use.

d) Find the probability that two randomly chosen cartridges will both have to be replaced before each has been used for 600 hours.

7) For the following continuous random variables X, with the given probability density functions, $f(x)$, find the median value of X. (It might help if you sketch the relevant graphs.)

{Note that for *simple* graphs you may not require integration! However, if using integration, then be careful with your *limits*. }

a) $f(x) = \frac{3}{8}x^2$ for $0 \le x \le 2$.

b) $f(x) = \frac{1}{5}$ for $-1 \le x \le 4$. **{You do not need integration with this one!}**

c)
$$f(x) = \begin{cases} \left(1 - \frac{1}{2}x\right) & \text{for } 0 \le x \le 2, \\ 0 & \text{otherwise.} \end{cases}$$

d) $f(x) = \frac{3}{56}(x+2)^2$ for $0 \le x \le 2$.

8) The life, X, of a certain brand of lightbulb is modelled by the probability density function

$$f(x) = \begin{cases} 2e^{-2x} & \text{for } x \ge 0, \\ 0 & \text{otherwise,} \end{cases}$$

where X is measured in thousands of hours.

Find the median life span of the lightbulbs.

9) A computer ink cartridge has a life of X hours. The variable X is modelled by the probability density function

$$f(x) = \begin{cases} 400x^{-2} & \text{for } x \ge 400, \\ 0 & \text{otherwise.} \end{cases}$$

Find the median life of the ink cartridges.

10) For the following continuous random variables X, with the given probability density functions, $f(x)$, find the expected value and variance of X.

a) $f(x) = \frac{3}{8}x^2$ for $0 \le x \le 2$.

b) $f(x) = \frac{1}{5}$ for $-1 \le x \le 4$.

c)
$$f(x) = \begin{cases} \left(1 - \frac{1}{2}x\right) & \text{for } 0 \le x \le 2, \\ 0 & \text{otherwise.} \end{cases}$$

d) $f(x) = \frac{3}{56}(x+2)^2$ for $0 \le x \le 2$.

11) A computer ink cartridge has a life of X hours. The variable X is modelled by the probability density function

$$f(x) = \begin{cases} 720x^{-2} & \text{for } 400 \le x \le 900, \\ 0 & \text{otherwise.} \end{cases}$$

a) Sketch the graph of $y = f(x)$.

b) Find the mean and variance of the lives of these cartridges.

12) The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} kx^2(3-x) & \text{for } 0 \le x \le 3, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

i) Show that $k = \frac{4}{27}$, and find E[X].

ii) Find P($X < 2$).

iii) Use your answer to part ii) to state, with a reason, whether the median of X is less than 2, equal to 2 or greater than 2.

13) The time, in minutes, between two consecutive calls to a telephone switchboard is modelled by a continuous random variable, X. The probability density function, $f(x)$, for this random variable is given by

$$f(x) = \begin{cases} k(10-x) & \text{for } 0 \le x \le 10, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

a) Calculate the value of k.

b) Find the mean time, E[X], between two consecutive calls.

c) Find Var[X].

14) A continuous random variable, X, has the probability density function

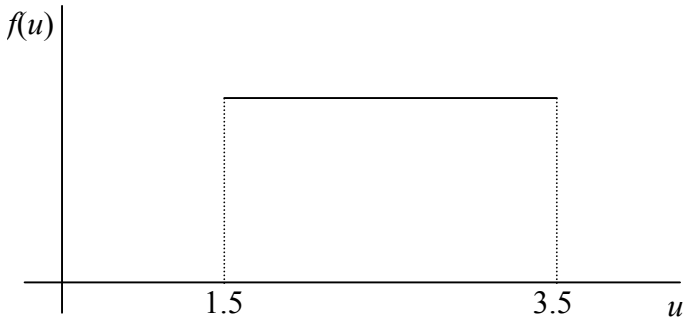
$$f(x) = \begin{cases} \frac{1}{2}x & \text{for } 0 \le x \le 2, \\ 0 & \text{otherwise.} \end{cases}$$

a) Find the median value of X.

b) Find E[X].

c) Find Var[X].

15) A continuous random variable, U, is uniformly distributed on $1.5 \le u \le 3.5$, as shown in the diagram.



a) Find the probability density function $f(u)$.

b) State the mean of U. {i.e. do NOT calculate the mean, the answer should be obvious!}

c) Use integration to calculate the variance of U.

16) The random variable, X, has the probability density function

$$f(x) = \begin{cases} kx^3 & \text{for } 0 \le x \le 2, \\ 0 & \text{otherwise.} \end{cases}$$

a) Find the value of k.

b) Find E[X].

c) Find Var[X].

d) Find the median of the distribution.

e) Find the probability that an observation lies within one **standard deviation** of the mean.
{Recall that the standard deviation is the square-root of the variance etc.}

ANSWERS.

1) a) $k = \frac{3}{8}$, c) $\frac{7}{8}$, d) $\frac{13}{32}$.

2) b) $k = \frac{1}{5}$, c) $\frac{2}{5}$.

3) a) $c = 1$, b) $\frac{1}{4}$.

4) a) $k = \frac{3}{56}$, c) $\frac{19}{56}$, $\frac{37}{56}$.

5) a) $k = 2$, b) 0.135, c) 0.632.

6) a) $k = 400$, b) $\frac{4}{5}$, c) $\frac{1}{3}$, d) $\frac{1}{9}$.

7) a) $\sqrt[3]{4}$, b) 1.5, c) 0.58578644, d) 1.30192725.

8) 500ln 2 hours. {Remember that X is in thousands of hours!}

9) 800 hours.

10) a) $E[X] = \frac{3}{2}$, $Var[X] = \frac{3}{20}$, b) $E[X] = \frac{3}{2}$, $Var[X] = \frac{25}{12}$, c) $E[X] = \frac{2}{3}$, $Var[X] = \frac{2}{9}$,
d) $E[X] = \frac{17}{14}$, $Var[X] = \frac{291}{980}$.

11) b) $E[X] = 584$, $Var[X] = 19100$.

12) i) $\frac{9}{5}$, ii) $\frac{16}{27}$, iii) the median is less than 2 (why?).

13) a) $k = \frac{1}{50}$, b) $\frac{10}{3}$, c) $\frac{50}{9}$.

14) a) $\sqrt{2}$, b) $\frac{4}{3}$, c) $\frac{2}{9}$.

15) a) $f(u) = \frac{1}{2}$ for $1.5 \le u \le 3.5$, b) $E[U] = 2.5$, c) $Var[U] = \frac{1}{3}$.

16) a) $k = \frac{1}{4}$, b) 1.6, c) 0.107, d) 1.68, e) 0.697.