COMPLEX NUMBERS

1) Sketch the following loci in separate Argand diagrams. Any relevant points or directions should be clearly indicated.

a) $|z| = 5$, b) $|z - 2i| = 5$, c) $|z - 4| \leq 2$,
d) $|z - 3 + 4i| = 5$, e) $|z + 12 - 5i| = 13$, f) $\arg(z) = \frac{\pi}{4}$,
g) $\arg(z - 2 - 3i) = \frac{\pi}{4}$, h) $\arg(z + 5 - 3i) = \frac{\pi}{6}$, i) $|z - 2| = |z - 2i|$,
j) $|z - 2| < |z - 2i|$, k) $|z + 3 - 2i| = |z - 5 + 3i|$.

2) A complex number z satisfies $|z - 3 - 4i| = 2$.
Describe in geometrical terms, with the aid of a sketch, the locus of the point which represents z in an Argand diagram.

Find, i) the greatest value of $|z|$,
ii) the difference between the greatest and least values of $\arg(z)$.

3) Repeat question 2) for $|z - 5 - 12i| = 4$.

4) In an Argand diagram, the point *P* represents the complex number z. On a single diagram, illustrate the set of possible positions of *P* for each of the following cases. Any relevant points or directions should be clearly indicated.

i) $|z - 3i| \leq 3$, ii) $\arg(z + 3 - 3i) = \frac{\pi}{4}$.

Given that z satisfies both i) and ii), find the greatest possible value of $|z|$.

5) It is given that $3 - i$ is a root of the quadratic equation $z^2 - (a + bi)z + 4(1 + 3i) = 0$, where *a* and *b* are both real.

In either order,
i) find *a* and *b*,
ii) find the other root of the equation, given that it is of the form *k*i, where *k* is real.

6) Given that z is a complex number such that $z + 3z^* = 12 + 8i$, find z, giving your answer in the form $x + iy$. **{Hint:** substitute $z = x + iy$.**}**

7) Given that $|z - 5| = |z - 2 - 3i|$, show on an Argand diagram the locus of the point which represents z. Any relevant points or directions should be clearly indicated.

Using your diagram, show that there is no value of z satisfying both
$|z - 5| = |z - 2 - 3i|$ and $\arg(z) = \frac{\pi}{4}$. **{Hint:** the locus of z is a straight line, find its gradient!**}**

8) Given that $(5 + 12i)z = 63 + 16i$, express z in the form $a + bi$ and hence find $|z|$ and $\arg(z)$.

Given also that $w = 3\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$, find i) $\left|\frac{z}{w}\right|$ and ii) $\arg(zw)$.

ANSWERS.

2) Circle centre (3, 4) radius 2 units.
i) Greatest $|z| = 7$. ii) 0.824 radians.

3) Circle centre (5, 12) radius 4 units.
i) Greatest $|z| = 17$. ii) 0.626 radians.

4) Greatest $|z| = 6$.

5) i) $a = 3$, $b = 3$.
ii) 4i.

6) $z = 3 - 4i$.

8) $z = 3 - 4i$. $|z| = 5$, $\arg(z) = -0.927$ radians.
i) $\frac{5}{3}$.
ii) 1.974 radians.