ARRANGEMENTS

1) If everyone had only two names, a Christian name and a surname, how many distinct pairs of initials would be possible?

2) A manufacturer supplies a model car in 6 colours and with 4 optional extras: radio, rear windscreen wiper, reversing lights and fluffy dice. How many distinct types of car may be ordered?

3) A four-volume dictionary is replaced on a shelf. In how many different orders may the volumes be replaced?

4) How many distinct rearrangements of the letters COWPAT are possible?

5) a) How many distinct rearrangements of the letters FLUTE are possible?

b) How many distinct rearrangements of the letters FLUTE are possible if the two vowels must be together?

6) Six people, including three friends, are to be stood in a line. In how many ways can this be done

a) if there are no restrictions,
b) if the three friends are to be placed together?

State the probability that for a group of 6 people, including 3 friends, who are stood in a line in a random order, the 3 friends are stood together

7) The letters A B C D E are to be placed in a line in a random order.

What is the probability that the resulting word ends in the letter E ?

8) In how many ways can the letters of the word SLEAFORD be arranged in a line ?

What is the probability that a random arrangement begins with S and ends with D ?

9) Six people are to be arranged in a line. What is the probability that the two youngest people are stood together ?

10) On a shelf there are 3 Mathematics books and 4 English books.

a) If the books are to be arranged so that all the Mathematics books are together, in how many ways can this be done ?

b) What is the probability that all the Mathematics books will not be together ?

11) Four boys and two girls sit in a line on stools in front of a coffee bar.

a) In how many ways can they arrange themselves so that the two girls are together ?

b) In how many ways can they sit if the two girls are not together ?

12) Twelve horses run in a race. The published results list the horses finishing first, second and third. Assuming there are no dead-heats, find the number or different possible published results.

13) A computer terminal displaying text can generate 16 different colours numbered 1 to 16. Any one of colours 1 to 8 may be used as the "background colour" on the screen, and any one of colours 1 to 16 may be used as the "text colour"; however, selecting the same colour for background and text renders the text invisible and so this combination is not used.
Find the number of different usable combinations of background colour and text colour.

14) How many distinct arrangements are there of the words

a) SOOTY, b) CAMERA, c) ONTOLOGY.

15) The digits of the number 314152 are rearranged to make different numbers.

a) How many different numbers can be formed? **{Hint: take care with the two 1's !}**

b) How many different ODD numbers can be formed?

16) The digits of the number 165211 are rearranged to make different numbers.

How many different ODD numbers can be formed.

17) Find the number of different arrangements of the letters of the word ELEVEN.

Find the number of different arrangements of the letters of the word ELEVEN if the three E's are to be consecutive.

18) The digits of the number 21352 are rearranged to make different numbers.

a) How many different numbers can be formed?

b) How many different numbers can be formed which are multiples of 5?

19) If a four digit number is formed from the digits 1, 2, 3 and 5 and repetitions are not allowed, find the probability that the number is divisible by 5.

20) Find the number of ways of arranging the letters of the word NEEDLESS.

21) Find how many four digit numbers can be formed from the digits 2, 3, 4, 5, 7 and 9 without repeating any digit.

Find also how many of these are a) less than 7000,
b) odd.

22) How many permutations are there of the letters of the word PARALLELOGRAM ?

23) How many distinct numbers can be formed from the digits 1, 2, 3, 4, 5, 6, 7 if no two even or no two odd digits are next to each other ?

24) How many distinct numbers can be formed from the digits 1, 2, 3, 4, 5, 6 if no two even or no two odd digits are next to each other ?

25) Six people are seated on a bench. How many arrangements are there if the tallest and shortest persons are to be separated ?

26) a) How many different 7-digit numbers can be formed from the digits 0, 1, 2, 2, 3, 3, 3 assuming that a number cannot start with a 0 ?

b) How many of these numbers will end in 0 ?

*27) The letters of the word POSSESSES are written on 9 cards, one on each card. The cards are shuffled and four of them are selected and arranged in a straight line.

How many possible arrangements are there of 4 letters ?

**{Hint: You must count the arrangements in some systematic way.
One way would be to count the number of arrangements involving 3 S's, then count the numbers involving 2 S's etc.}**

28) Show that ${}^{12}C_3 = 220$.

29) Show that ${}^{20}C_5 = 15504$

30) A university entrance exam consists of 10 questions from which the applicant is to select any 7 questions to attempt. How many possible sets of 7 questions are there for the applicant to attempt?

31) A bookshelf contains 20 books in total. Jenny wishes to select 3 books for a study. In how many ways can she do this ?

32) Of the 25 people in a certain class, 6 are to be chosen to complete a questionnaire. How many possible samples of 6 students are there?

33) From a normal pack of 52 playing cards, a hand of 4 cards is to be dealt at random.

a) How many possible hands are there?

b) What is the probability that a randomly dealt hand of 4 cards comprises the 4 aces?

34) 15 players are available for a hockey team of 11 players. How many possible teams could be selected?

35) Find the number of ways in which a team of 3 women and 2 men can be selected from a group of 6 women and 7 men.

36) From a normal pack of 52 playing cards, a hand of 4 cards is to be dealt at random.

How many hands are there which contain exactly two aces ?

{Hint: there are ${}^{4}C_{2}$ ways of choosing 2 aces and ${}^{48}C_{2}$ ways of selecting the remaining 2 cards etc.}

37) In how many ways can a committee of 5 people be selected from 8 men and 4 women if it must contain:

a) 3 men and 2 women, b) at least 2 women?

38) Three boys and five girls are to be seated on a bench so that the youngest boy and the youngest girl sit next to one another. In how many ways can this be done?

{Hint: permutations or combinations?}

39) How many numbers greater than 50000 can be formed from the digits 2, 3, 4, 5,6 if each digit is used **exactly** once in each number?

40) In how many ways can a committee of 5 people be selected from 7 men and 3 women if it must contain:

a) 3 men and 2 women, b) 3 women and 2 men,

c) at least one woman?

41) Ten people travel in two cars, a saloon and a Mini. If the saloon has seats for six and the Mini has seats for four, find the number of different ways in which the party can travel, assuming that the order of seating in each car does not matter and all the people can drive.

{Take care with this, it is simple – but more subtle than you might think!}

42) A class contains 30 children, 18 girls and 12 boys. Four complementary theatre tickets are distributed at random to the children in the class. What is the probability that

a) all four tickets go to the girls,

b) two boys and two girls receive tickets?

43) Eight different cards, of which four are red and four are black, are dealt to two players so that each receives a hand of four cards. Calculate

a) the total number of different hands which a given player could receive,

b) the probability that each player receives a hand consisting of four cards all of the same colour.

44) Nine people are to be seated at three tables holding 2, 3 and 4 people respectively. In how many ways can the groups sitting at the tables be selected, assuming that the order of sitting at the table does not matter?

45) A piece of wood of length 10 cm is to be divided into 3 pieces so that the length of each piece is a whole number of cm, for example 2 cm, 3 cm and 5 cm.

a) List all the different sets of lengths which could be obtained.

b) If one of these sets is selected at random, what is the probability that the lengths of the pieces could be lengths of the sides of a triangle?
{Hint: for example; 1 cm, 1 cm, 8 cm could not form a triangle! Why not?}

46) Calculate the number of ways in which three girls and four boys can be seated on a row of seven chairs if each arrangement is to be symmetrical.

{Hint: tricky! You will need to list out some possibilities. There must be a girl in the middle seat etc.}

*47) Calculate the number of different ways in which the letters of the word *TRIANGLES* can be arranged if no two vowels may come together.

{Very tricky! Easiest way is to think about where the vowels can be positioned relative to the consonants etc.}

Answers.

1) 676. 2) 24. 3) 24. 4) 720.

5) a) 120, b) 48. 6) a) 720, b) 144. $\frac{1}{5}$.

7) $\frac{1}{5}$. 8) 40320. $\frac{1}{56}$. 9) $\frac{1}{3}$.

10) a) 720, b) $\frac{1}{7}$.

11) a) 240, b) 480.

12) 1320.

13) 120.

14) a) 60, b) 360, c) 6720.

15) a) 360, b) 240.

16) 80.

17) 120. 24.

18) a) 60, b) 12.

19) $\frac{1}{4}$.

20) 3360.

21) 360. a) 240, b) 240.

22) 86486400.

23) 144.

24) 72.

25) 480.

26) a) 360, b) 60.

27) 115.

30) 120.

31) 1140.

32) 177100.

33) a) 270725, b) $\frac{1}{270725}$.

34) 1365.

35) 420.

36) 6768.

37) a) 336, b) 456.

38) 10080.

39) 48.

40) a) 105, b) 21, c) 231.

41) ${}^{10}C_6$ or ${}^{10}C_4 = 210$.

42) a) $\frac{68}{609}$, b) $\frac{374}{1015}$.

43) a) 70, b) $\frac{1}{35}$.

44) 1260.

45) a) (1, 1, 8), (1, 2, 7), (1, 3, 6), (1, 4, 5), (2, 3, 5), (2, 4, 4), (2, 6, 2), (3, 3, 4).

b) $\frac{1}{4}$.

46) 432.

47) 151200.