

ARRANGEMENTS

AS LEVEL EXAM QUESTIONS

1. Seven men and five women have been nominated to serve on a committee. The committee consists of four members who are to be chosen from the seven men and five women.
 - i) In how many different ways can the committee be chosen?
 - ii) In how many of these ways will the committee consist of two men and two women?
 - iii) Given that every possible committee of four is equally likely, calculate the probability that the chosen committee consists of exactly two men.

2. Three married couples, Mr and Mrs Aziz, Mr and Mrs Baker and Mr and Mrs Campbell, are arranged in a line for a photograph.
 - i) How many different arrangements of the six people are possible?
 - ii) In how many of these arrangements is Mr Aziz standing next to his wife?
 - iii) Given that every possible arrangement is equally likely, calculate the probability that Mr Aziz is standing next to his wife.

3. A standard pack of playing cards consists of 52 distinct cards. Five different cards are selected at random. The order in which the cards are selected does not matter.
 - i) Find the number of different possible selections of 5 cards.
 - ii) There are 4 suits (clubs, diamonds, hearts and spades) and each suit consists of 13 cards. How many of the selections in part i) consist of 3 spades and 2 clubs?
 - iii) How many of the selections in part i) contain exactly 3 spades?
{Hint: we require 3 spades and 2 cards from all the remaining cards.}
 - iv) Calculate the probability that 5 cards selected at random will consist of 3 spades and 2 clubs.

4. A child's game uses five bricks. One is blue, one is green, one is yellow and two are white. The five bricks are arranged in a line.
 - i) How many different possible arrangements of the colours are there?
{Take care with the repetitions!}
 - ii) Assuming that all the arrangements in part i) are equally likely, find the probability that the two white bricks are at the ends of the line.

5. Each of 6 cards has a different single letter written on it. The letters on the cards are A , B , C , D , E and F . The cards are shuffled and then placed in a row.
- How many different possible arrangements of letters are there?
 - In how many of these arrangements are the vowels (i.e. the letters A and E) next to each other?
 - The cards are now shuffled and placed face down. Three of the cards are selected at random. Find the probability that at least one of the selected cards is a vowel.
{Hint: draw a tree diagram which lists; 'vowel', 'not vowel' etc.}
6. Three families, Mr and Mrs Dale and their son, Mr and Mrs Mason and their daughter, and Mr and Mrs Baker and their three children, are going to the theatre together. They have booked seats in one row which contains exactly eleven seats.
- If there is no restriction on seating, state the number of different possible arrangements of the 11 people when they sit in the row.
 - If each family has to sit together, how many arrangements are possible?
 - If the seating arrangement is chosen at random, find the probability that each family is sitting together.

ANSWERS.

1. i) ${}^{12}C_4 = 495.$
ii) ${}^7C_2 \times {}^5C_2 = 210.$
iii) $\frac{210}{495} = \frac{14}{33}.$
2. i) $6! = 720.$
ii) $5 \times 2! \times 4! = 240.$
iii) $\frac{240}{720} = \frac{1}{3}.$
3. i) ${}^{52}C_5 = 2598960.$
ii) ${}^{13}C_3 \times {}^{13}C_2 = 22308.$
iii) ${}^{13}C_3 \times {}^{39}C_2 = 211926.$
iv) $\frac{22308}{2598960} = 0.00858\dots$
4. i) $\frac{5!}{2!} = 60.$
ii) Number of arrangements with the white bricks at the ends = $3! = 6.$
Probability therefore = $\frac{6}{60} = \frac{1}{10}.$
5. i) $6! = 720.$
ii) $5 \times 2! \times 4! = 240.$
iii) $P(\text{At least one vowel}) = 1 - P(\text{no vowels}) = 1 - \frac{4}{6} \times \frac{3}{5} \times \frac{2}{4} = \frac{4}{5}.$
6. i) $11! = 39916800.$
ii) $3! \times (3! \times 3! \times 5!) = 25920.$
iii) $\frac{25920}{39916800}.$