LINEAR PROGRAMMING: the Simplex algorithm.

Step 1.	Formulate the maximising problem as a tableau.
Step 2.	Ensure that all elements in the value column (except maybe the top one) are non-negative.
Step 3.	Select any column (pivotal column) except the value column of the tableau whose entry in the objective function row is negative.
Step 4.	Divide each entry in the value column by corresponding **positive** entries in the pivotal column. Select the row with the smallest resulting value. The entry in the selected row of the pivotal column is called the **pivot**.
Step 5.	Divide every entry in the selected row by the pivot.
Step 6.	Combine appropriate multiples of the selected row with all the other rows in order to reduce to zero all other entries in the pivotal column.
Step 7.	If all entries in the objective row (except possibly the last one) are non-negative then the maximum has been reached. Otherwise return to step 3.
Step 8.	The last column contains the values of the objective function and the non-zero variables.

1) Maximise $2x + 3y$ subject to the constraints $x + 2y \leq 6$, $x + y \leq 5$ and $x, y \geq 0$, using:
a) a graphical method, b) the Simplex method.

2) A small business makes two products *A* and *B*.
Each item of article *A* takes 1 hour of machine time and 1 hour of manual labour to finish.
Each item of article *B* takes 2 hours of machine time and 1 hour of manual labour to finish.

One day there are 6 hours of machine time and 5 hours of manual time available.

The profit on each item of article *A* is £2 and each item of article *B* is £1.

i) Set up the problem of maximising the daily profit as a linear programming problem.

ii) By introducing slack variables, formulate this problem as an initial Simplex tableau.

iii) Solve the linear programming problem by the Simplex method.

3) Maximise $9x + 4y$ subject to the constraints $3x + 4y \leq 48$, $2x + y \leq 17$, $3x + y \leq 24$ and $x, y \geq 0$, using:
a) a graphical method, b) the Simplex method using 3 slack variables, *u*, *v* and *w*.

4) Maximise $F = -x + 8y + z$ subject to the constraints $x + 2y + 9z \leq 10$, $y + 4z \leq 12$ and $x, y \geq 0$, using the Simplex method.

5) Maximise $F = 20x + 30y$ subject to the constraints $7x + 10y \le 80$, $x + 2y \le 12$ and $x, y \ge 0$, using the Simplex method.

6) Maximise $F = 10x + 7y - 50$ subject to the constraints $3x + 2y \le 42$, $2x + 5y \le 50$ and $x, y \ge 0$, using the Simplex method. **{Hint: write the objective row as $F - 10x - 7y = -50$ etc.}**

7) A particular linear programming problem can be summarised thus:

Maximise $P = 4000x + 6000y$
subject to $x + y \le 5000$, $-x + 4y \le 0$, $x + 3y \le 6000$ and $x, y \ge 0$.

a) By introducing slack variables u, v and w, set up the problem as an initial Simplex tableau.

b) Perform one iteration on the tableau, beginning by choosing to pivot on the x-column.

c) State the values of x, y and P resulting from the iteration in b).

d) Explain how you know, from the tableau, that the optimal solution has not yet been reached.

8) Three processes, I, II and III, are involved in the manufacture of three products, A, B and C. For each product, the manufacturing time (hours) and profits per item (£) are shown.

Product	I	II	III	Profit
A	1	2	3	120
B	5	1	2	70
C	4	4	1	100

The total available manufacturing times on processes I, II and III are 90, 35 and 60 hours respectively.

a) What mix of products yields the greatest profit?
b) What assumptions have you made in answering part a).

9) Consider the linear programming problem.

Maximise $P = 2x + y$
subject to $x + y \le 7$, $x + 2y \le 10$, $2x + 3y \le 16$, $x, y \ge 0$.

a) By introducing slack variables u, v and w, set up the problem as an initial Simplex tableau.

b) Perform one iteration on the tableau, beginning by choosing to pivot on the x-column.

c) State the values of x, y and P resulting from the iteration in b).

d) Explain how you know whether or not the optimal solution has been achieved.

10) Minimise $x + y - 2z$ subject to the constraints $2x + y \geq z$, $2x + 5 \geq 3z$, $3y + 4z \leq 12$ and $x, y, z \geq 0$ using the Simplex method. **{Maximise the negative quantity etc.}**

{Hint: remember that slack variables must be <u>positive</u>.

E.g. the constraint $2x + y \geq z$.

First write this as $2x + y - z \geq 0$. Now introduce a slack variable u and write $2x + y - z - u = 0$ in order that $u \geq 0$ etc.}

ANSWERS.

1) Max value = 11 when $x = 4, y = 1$; b) 2 iterations required.

2) i) x = number of item A, y = number of item B, maximise $P = 2x + y$ subject to $x + 2y \leq 6, x + y \leq 5, x, y \geq 0$, ii) $P - 2x - y = 0$, $x + 2y + s = 6$, $x + y + t = 5$, iii) max $P = 10$ when 5 of item A and 0 item B are made. 1 iteration needed.

3) Max value = 75 when $x = 7$ and $y = 3$; b) 2 iterations required.

4) Max value of $F = 40$ when $x = 0, y = 5$ and $z = 0$. 1 iteration required.

5) Max value of $F = 230$ when $x = 10$ and $y = 1$. 2 iterations required.

6) Max value of F is 92 when $x = 10$ and $y = 6$. 2 iterations are required.

7) a)

	P	x	y	u	v	w	Value	
<u>1</u>	1	−4000	−6000	0	0	0	0	
2	0	**1**	1	1	0	0	5000	
3	0	−1	4	0	1	0	0	
4	0	1	3	0	0	1	6000	

b) Pivot = second entry in the x column.

	P	x	y	u	v	w	Value	
<u>5</u>	1	0	−2000	4000	0	0	20000000	**5** = **1** + 4000 × **6**.
6	0	1	1	1	0	0	5000	**6** = **2**.
7	0	0	5	1	1	0	5000	**7** = **3** + **6**.
8	0	0	2	−1	0	1	1000	**8** = **4** – **2**.

c) This shows that $P = 20000000$ when $x = 5000$ and $y = 0$.

d) The optimal solution has not been reached because the objective row contains a negative value in the y column.

8) a) 10 of A, 15 of B and 0 of C; 2 iterations are required, b) assumptions are the all items will be sold and that there are no existing contracts to make item C.

9) a)

	P	x	y	u	v	w	Value	
<u>1</u>	1	−2	−1	0	0	0	0	
2	0	**1**	1	1	0	0	7	
3	0	1	2	0	1	0	10	
4	0	2	3	0	0	1	16	

b)

	P	x	y	u	v	w	Value	
<u>5</u>	1	0	1	2	0	0	14	**5** = **1** + 2 × **6**.
6	0	1	1	1	0	0	7	**6** = **2**.
7	0	0	1	−1	1	0	3	**7** = **3** – **6**.
8	0	0	1	−2	0	1	2	**8** = **4** – 2 × **6**.

c) This shows that $P = 14$ when $x = 7$ and $y = 0$.

d) The optimal solution has been reached since there are no negative entries in the objective row.

10) Min value = −4 when $x = 2, y = 0$ and $z = 3$.