LINEAR PROGRAMMING – GRAPHICAL SOLUTIONS

1) Solve, graphically, the following linear programming problems.

a) Maximise $2x + y$ subject to the constraints:

$x + y \leq 6$
$x \leq 5$
$y \leq 4$
$x \geq 0$
$y \geq 0$.

b) Maximise $x + 5y$ subject to the constraints:

$4x + 3y \leq 12$
$2x + 5y \leq 10$
$x \geq 0$
$y \geq 0$.

c) Maximise $3x + 2y$ subject to the constraints:

$y + 2x \leq 12$
$x \geq 2$
$y \geq 4$.

d) Minimise $2x + y$ subject to the constraints:

$3x + y \geq 6$
$x + y \geq 4$
$x \leq 3$
$y \leq 4$.

e) Maximise $2x + 3y$ subject to the constraints:

$3x + y \leq 30$
$x + 2y \leq 30$
$x \geq 0$
$y \geq 0$.

f) Minimise $12u + 10v$ subject to the constraints:

$2u + 3v \geq 16$
$4u + 2v \geq 24$
$u \geq 0$
$v \geq 0$.

For the following questions, always ask yourself if <u>integer solutions</u> are required because a question may not explicitly make this clear. The context of the question should make it clear however.

2) A builder has a plot of land available on which he can build houses. He can build either luxury houses or standard houses. He decides to build at least five luxury houses and at least 10 standard houses. Planning regulations prevent him from building more than 30 houses altogether.

Each luxury house requires 300 m^2 of land and each standard house requires 150 m^2 of land. The total area of plot is 6000 m^2.

The builder makes a profit of £12 000 on each luxury house and a profit of £8000 on each standard house. The builder wishes to make the maximum possible profit.

a) Letting x stand for the number of luxury houses and y stand for the number of standard houses, formulate the above problem as a linear programming problem.

b) Determine how many of each type of house he should build to make the maximum profit and state what this profit is.

3) KJB haulage receives an order to transport 1600 packages. They have large vans, which can take 200 packages each, and small vans, which can take 80 packages each.

The cost of running each large van on the required journey is £40 and the cost of running each small van on the same journey is £20.
There is a limited budget for the job which requires that not more than £340 be spent. It is additionally required that the number of small vans used must not exceed the number of large vans used.

a) Letting x stand for the number of large vans and y stand for the number of small vans, show that the requirement that between them, the various vans must transport at least 1600 packages leads to the inequality $5x + 2y \geq 40$.

b) Formulate the above problem as a linear programming problem in order to minimise costs. **{Hint: simplify inequalities.}**

c) How many of each kind of van should be used if costs are to be kept to a minimum?

4) A minibus operator is contracted to transport 50 workers to their place of employment. He has available three type A minibuses and four type B minibuses.

A type A minibus carries 15 workers and a type B minibus carries 10 workers. Only five drivers are available. It costs £50 to operate a type A minibus and £40 to operate a type B minibus.

Let x be the number of type A minibuses used and y the number of type B minibuses used.

a) Write down four inequalities satisfied by x and y, other than $x \geq 0$, $y \geq 0$.

b) Display these inequalities on a graph and label clearly the feasible region.

Given that x and y must be integers:

c) determine the possible combinations of x and y that satisfy the constraints,

d) determine the minimum cost of the operation and the numbers of type A and type B minibuses used in this case.

5) A factory produces two items. Each day there are 160 labour hours available and 200 machine hours. The details are summarised in the table below.

	Item 1	Item 2	Availability
Labour hours	2 hours	4 hours	160 hours
Machine hours	5 hours	2 hours	200 hours

{For example, item 1 requires 2 hours of labour and 5 hours of machine time.}

a) Formulate the problem of maximising the total daily output of items as a linear programming problem.

b) How should the production be divided between the two items if the total daily output is to be maximised?

c) Suppose the profit on every item 1 is £50 and £10 on every item 2. How should the production be divided if we now maximise the daily profit?
{Hint: use the same feasible region as b) but change the objective function to give profit, not total number of items produced and check each vertex etc.}

6) When a lecturer retires he will have up to £30 000 to invest. He decides to invest some of the money in stocks with a 7% yield per annum and some of the money in a bond with a 5% yield per annum.

He decides that no more than £20 000 shall be invested in either option alone.

How much should he invest in each option to maximise his yield?

ANSWERS.

1) a) 11.
b) 10.
c) 22.
d) 5.
e) 48.
f) 80.

2) a) Maximise $12000x + 8000y$ subject to $x \geq 5$, $y \geq 10$, $x + y \leq 30$, $300x + 150y \leq 6000$, x and y must be integers.
b) 10 luxury homes, 20 standard homes; profit = £280 000.

3) b) Minimise $40x + 20y$ subject to: $5x + 2y \geq 40$, $2x + y \leq 17$, $y \leq x$, $y \geq 0$, x and y must be integers.
c) 8 large vans, 0 small vans; cost = £320.

4) a) $x \leq 3$, $y \leq 4$, $x + y \leq 5$, $15x + 10y \geq 50$ or $3x + 2y \geq 10$.
c) (1, 4), (2, 2), (2, 3), (3, 1), (3, 2).
d) Min. cost £180 when 2 of each type of minibus are used.

5) a) x = number of item 1, y = number of item 2; maximise $x + y$ subject to $2x + 4y \leq 160$ or $x + 2y \leq 80$, $5x + 2y \leq 200$, $x \geq 0$, $y \geq 0$, x and y must be integers.
b) make 30 of item 1 and 25 of item 2.
c) make 40 of item 1, 0 of item 2.

6) Suppose £x is invested in stock and £y is invested in the bond. Maximise $0.07x + 0.05y$ subject to $x + y \leq 30\,000$, $x \leq 20\,000$, $y \leq 20\,000$, $x \geq 0$, $y \geq 0$. Invest £20 000 in stock, £10 000 in the bond and max. yield = £1 900.