DIFFERENTIAL EQUATIONS; Integrating factors

$\frac{dy}{dx} + P(x)y = Q(x)$; use the integrating factor $e^{\int P(x)dx}$.
Remember to simplify an integrating factor as far as possible!

For questions 1) - 14), find the general solutions of the differential equations.

1) $\frac{dy}{dx} + 3y = e^{-3x}$

2) $x\frac{dy}{dx} + y = 1$

3) $\frac{dy}{dx} + y\cot x = \operatorname{cosec} x$

4) $x^2\frac{dy}{dx} + xy = x + 1$

5) $\frac{dy}{dx} - \frac{3y}{x+1} = (x+1)^4$

6) $\tan x\frac{dy}{dx} + y = \cot x \tan x$

7) $x\frac{dy}{dx} = y + x^2(\sin x + \cos x)$

8) $x\frac{dy}{dx} = y - x^2e^{-x}$

9) $x\frac{dy}{dx} + (x+1)y = 2$

10) $x\frac{dy}{dx} + 2y = \frac{\sin x}{x}$

11) $\frac{dy}{dx} + y\sin x = \sin x$

12) $\frac{dy}{dx} = x - xy$

13) $x\frac{dy}{dx} + 3y = \frac{e^x}{x^2}$

14) $\frac{dy}{dx} - y\tan x - \sec x = 0$.

15) i) Use integration by parts to find $\int x\sin x\,dx$.

ii) Find the general solution of the differential equation $\frac{dy}{dx} + y\cot x = x$.

Find also the particular solution for which $y = 0$ when $x = \frac{1}{2}\pi$.

16) The general solution of the differential equation $x\frac{dy}{dx} + (x+1)y = 1$ is represented by a family of curves.

i) Find the general solution of the differential equation.

ii) Find the equation of the particular curve which passes through the point (1, 2).

17) Given that $-1 < x < 1$, find the general solution of the differential equation

$$(1-x^2)\frac{dy}{dx} - xy - 1 = 0.$$

Find the particular solution for which $y = \frac{1}{2}\pi$ when $x = 0$.

18) i) Use double-angle formulae to express $\sin^2 x$ in terms of $\cos 2x$.

ii) Find the general solution of the differential equation $\frac{dy}{dx} + y\cot x = \sin x$.

<u>ANSWERS.</u>

Check your answers carefully because they might appear incorrect when compared with some of the answers here, but they could still be correct! It is a question of how you simplify your answers, particularly with respect to the arbitrary constant which I generally label 'A'.

1) $y = \dfrac{x + A}{e^{3x}}$.

2) $y = 1 + \dfrac{A}{x}$.

3) $y = \dfrac{x + A}{\sin x}$.

4) $y = 1 + \dfrac{\ln x + A}{x}$.

5) $y = (x + 1)^3 \left\{ \dfrac{x^2}{2} + x + A \right\}$.

6) $y = 1 + A\operatorname{cosec} x$.

7) $y = (A + \sin x - \cos x)x$.

8) $y = (e^{-x} + A)x$.

9) $y = \dfrac{2e^x + A}{xe^x}$.

10) $y = \dfrac{A - \cos x}{x^2}$.

11) $y = 1 + Ae^{\cos x}$.

12) $y = 1 + A\,e^{-\frac{x^2}{2}}$.

13) $y = \dfrac{e^x + A}{x^3}$.

14) $y = \dfrac{x + A}{\cos x}$.

15) i) $-x\cos x + \sin x + A$.

ii) General solution is $y = A\operatorname{cosec} x - x\cot x + 1$.
Particular solution is $y = -\operatorname{cosec} x - x\cot x + 1$.

16) i) $y = \dfrac{1 + Ae^{-x}}{x}$.

ii) $y = \dfrac{1 + e^{1-x}}{x}$.

17) General solution is $y = \dfrac{\sin^{-1}x + A}{\sqrt{1 - x^2}}$.

Particular solution is $y = \dfrac{\sin^{-1}x + \frac{1}{2}\pi}{\sqrt{1 - x^2}}$.

18) i) $\sin^2 x = \dfrac{1}{2}(1 - \cos 2x)$.

ii) $y = \dfrac{2x - \sin 2x + B}{4\sin x}$. {Careful with this one in that you might have to simplify your answer somewhat!}