

- 6) Given that $\tanh x = \frac{24}{25}$, use the identity $1 - \tanh^2 x \equiv \operatorname{sech}^2 x$ to show that $\cosh x = \frac{25}{7}$.
Hence find $\sinh x$ and deduce that $x = \ln 7$.
- 7) Express each of the following in logarithmic form:
a) $\sinh^{-1}\left(\frac{3}{4}\right)$, b) $\cosh^{-1} 2$, c) $\tanh^{-1}\left(\frac{1}{2}\right)$.
- 8) Prove the identity $\coth^{-1} x = \frac{1}{2} \ln\left(\frac{x+1}{x-1}\right)$ for $|x| > 1$. Hence find the exact value of $\coth^{-1} 2$.
- 9) Express $\operatorname{sech}^{-1} x$ in logarithmic form for $0 < x < 1$, simplifying your answer as far as possible.
[Hint: put $y = \operatorname{sech}^{-1} x$ then $\operatorname{sech} y = x$. Find $\cosh y$ and $\sinh y$ in terms of x and use the identity $\cosh y + \sinh y = e^y$.]
- 10) Given that $y = \sinh^{-1} 2$, find, in surd form, the exact value of $\cosh y$.
- 11) Solve the equation $2 \tanh^{-1}\left(\frac{x-2}{x+2}\right) = \ln 2$.
- 12) Solve the following equations, leaving your answers in terms of natural logarithms where appropriate. **{Remember, you can always use the e^x definitions etc!}**
a) $1 + 7\sinh x = 4\cosh^2 x$, b) $\cosh x - 1 = \sinh^2 x$, c) $2\cosh x = 5\sinh x$,
d) $3\operatorname{sech}^2 x + 4\tanh x + 1 = 0$, e) $\cosh(\ln x) - \sinh\left(\ln \frac{1}{2}x\right) = 1\frac{3}{4}$,
f) $4\tanh^2 x - \operatorname{sech} x = 1$.
- 13) Starting from the definitions of $\sinh x$ and $\cosh x$, show that
$$\sinh(A+B) = \sinh A \cosh B + \cosh A \sinh B.$$

Hence express $5\cosh x + 13\sinh x$ in the form $r\sinh(x+\alpha)$ where $r > 0$.
Use your results to solve the equation $5\cosh x + 13\sinh x = 12\sinh 2$.
- 14) Starting from the definitions of $\sinh x$ and $\cosh x$, show that
$$\cosh(A-B) = \cosh A \cosh B - \sinh A \sinh B.$$

Hence express $25\cosh x - 24\sinh x$ in the form $r\cosh(x-\alpha)$ where $r > 0$, showing clearly that $\alpha = \ln 7$.
Hence write down the minimum value of $25\cosh x - 24\sinh x$ and find the value of x at which this occurs, giving your answer in terms of natural logarithms.

ANSWERS.

3) a) $\frac{13}{12}$. b) $\frac{5}{13}$. c) $\frac{12}{13}$. d) $\frac{13}{5}$.
e) $\frac{65}{72}$. f) $\frac{97}{72}$.

4) $\pm \frac{3}{4}$.

5) a) $x = \ln 3$. b) $x = \frac{1}{2} \ln 3$. c) $x = 0$ or $x = \ln \frac{5}{3}$.
d) $x = \ln 4$. e) $x = \ln 2$ or $x = -\ln 4$. f) $x = \ln 2$ or $x = -\ln 3$.
g) $x = -\ln 2$ or $x = \ln 2$. h) $x = \ln 3$ or $x = \ln(1 + \sqrt{2})$. i) $x = 0$ or $x = -\ln 2$.
j) $x = -\ln 4$ or $x = \ln 5$.

6) $\sinh x = \frac{24}{7}$.

7) a) $\ln 2$. b) $\ln(2 + \sqrt{3})$. c) $\frac{1}{2} \ln 3$.

8) $\frac{1}{2} \ln 3$.

9) $\ln \left(\frac{1 + \sqrt{1 - x^2}}{x} \right)$.

10) $\sqrt{5}$.

11) $x = 4$.

12) a) $x = \ln(1 + \sqrt{2})$ or $x = \ln 2$. b) $x = 0$. c) $x = \frac{1}{2} \ln \left(\frac{7}{3} \right)$.
d) $x = -\frac{1}{2} \ln 5$. e) $x = 1$ or $x = 6$. f) $x = \pm \ln \left(\frac{4 + \sqrt{7}}{3} \right)$.

13) $5\cosh x + 13\sinh x = 12\sinh \left(x + \ln \frac{3}{2} \right)$.
 $x = 2 - \ln \frac{3}{2}$.

14) $25\cosh x - 24\sinh x = 7\cosh(x - \ln 7)$.
Min value = 7 when $x = \ln 7$.