GEOMETRIC DISTRIBUTIONS

1) An unfair coin is such that the probability of obtaining a head when it is tossed is $0 \cdot 6$. The coin is tossed repeatedly until the first head is obtained. Assuming that each toss is independent of all other tosses, find the probability that the first head is obtained on the third toss.

State the expected number of tosses in order to obtain the first head.

2) I sit in a café and watch people go by. For each person that goes by, the probability that I know their name is $0 \cdot 05$. Assuming independence, find the probability that the fifth person who goes by is the first person whose name I know, giving your answer correct to 3 decimal places.

3) For the purpose of sampling, a computer generates a series of random numbers, each of which is an integer in the range 1 to 1000 inclusive. Each such integer is equally likely to be generated, independently of other integers in the series. A number generated is "acceptable" if it lies in the range 1 to 278 inclusive. The first "acceptable" number is the R^{th} number generated.

i) State the distribution of R.

ii) Calculate the probability that $R = 3$.

iii) Find the mean and variance of R.

4) i) A fair coin is tossed until a head has been obtained. Find the probability that exactly 5 tosses are required.

ii) A fair coin is tossed until both a head and a tail have been obtained. Find the probability that exactly 5 tosses are required.

5) On a production line, 12% of completed items are faulty. Each completed item is tested. The number of items that have been tested when the first faulty item is found is X. Stating a necessary assumption, suggest an appropriate model for the distribution of X.

Using your model, find i) $P(X \geq 3)$, ii) $E[X]$.

6) The random variable Y takes values 1, 2, 3, … and has a geometric distribution. The probability that Y takes the value 1 is $0 \cdot 25$.

i) Find the probability that Y takes the value 3.

ii) Find the mean and variance of Y.

7) When I make a telephone call to an office, the probability of not getting through is $0 \cdot 45$. If I do not get through then I try again later. Let X denote the number of attempts I have to make in order to get through. Stating any necessary assumptions, identify the probability distribution of X.

Hence calculate i) $P(X \geq 4)$, ii) $E[X]$ and $Var[X]$.

iii) the smallest value of n, if there is to be at least a 95% chance of getting through on the phone on or before the n^{th} attempt.
{**HINT** can use trial and error for part iii).}

ANSWERS.

1) P(3rd toss) = 0.096.
E[Number of tosses] = $1\frac{2}{3}$.

2) 0.0407253.

3) i) $R \sim$ Geo(0.278).
ii) 0.1449169.
iii) E[R] = 3.5971223. Var[R] = 9.3421666.

4) i) 0.03125.
ii) 0.0625.

5) Assuming each item is independent of every other, $X \sim$ Geo(0.12).
i) 0.7744.
ii) $8\frac{1}{3}$.

6) i) 0.140625.
ii) E[Y] = 4. Var[Y] = 12.

7) Assuming successive attempts are independent of each other, $X \sim$ Geo(0.55).
i) 0.091125.
ii) E[X] = 1.818181… Var[X] = 1.4876.
iii) $n = 4$.