EXPECTATION and VARIANCE of discrete random variables

1) A sixteen–sided spinner consists of a regular sixteen–sided polygon with a pin through its centre. It has 2 edges labelled 1, three edges labelled 2, three edges labelled 3, four edges labelled 4 and four edges labelled 5.

Find the mean and variance of the score which results from one spin of the spinner.

2) A board game is played by moving a counter S squares forward at a time, where S is determined by the following rule.

> A fair six-sided die is thrown once. S is half the number shown on the die if that number is even; otherwise S is twice the number shown on the die.

i) Write out a table showing the possible values of S and their probabilities.

ii) Use your table to calculate the mean and variance of S.

3) A discrete random variable X has the probability distribution given in the following table:

x	1	3	5	7	9
$P(X = x)$	$\frac{1}{4}$	$\frac{1}{5}$	k	$\frac{3}{10}$	$\frac{1}{5}$

i) Find the value of the constant k.
ii) Find $P(X \geq 5)$.
iii) Find $E[X]$ and $Var[X]$.

4) A blue unbiased cubical die has one face marked 1, two faces marked 2 and three faces marked 3. A red unbiased cubical die has two faces marked 1, two marked 2 and two marked 3. The two dice are rolled together and X is the total score on the dice. Show that $P(X = 4) = \frac{1}{3}$, and draw up a table showing the probability distribution of X.

Find the mean of X.

5) A computer is programmed to produce a sequence of integers, R, in the range 0 to 5 inclusive, with probabilities as shown below.

r	0	1	2	3	4	5
$P(R = r)$	k	$\frac{1}{30}$	$\frac{2}{30}$	$\frac{3}{30}$	$\frac{4}{30}$	$\frac{5}{30}$

i) Show that $k = \frac{1}{2}$.

ii) Calculate the mean and variance of R.

6) A lottery machine selects single–digit numbers X between 0 and 9 inclusive. Unfortunately it is wrongly adjusted and the probability that $X = x$ is k if x is even but the probability is $2k$ if x is odd.

i) Show that $k = \frac{1}{15}$.

ii) Calculate the mean and variance of X.

7) In an arcade game, whenever a lever is pulled a number appears on a screen. The number can be 1, 2 or 3, and the probabilities of obtaining these numbers are given in the following table.

Number	1	2	3
Probability	0·1	0·5	0·4

i) The score, S_1, when the lever is pulled for the first time is the number obtained. Calculate the mean and variance of S_1.

ii) A second score, S_2, is obtained as follows. If $S_1 = 1$, the lever is pulled a second time, and S_2 is the number which then appears on the screen. If $S_1 = 2$ or $S_1 = 3$, the lever is not pulled a second time and S_2 is 0.

Tabulate the possible values of $S_1 + S_2$ and their associated probabilities.
Hence calculate the mean of $S_1 + S_2$.

ANSWERS.

1) Mean = $3\frac{5}{16}$, variance = $1\frac{215}{256}$.

2) i)

s	1	2	3	6	10
$P(S = s)$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

ii) $E[S] = 4$, $Var[S] = 9\frac{2}{3}$.

3) i) $k = \frac{1}{20}$, ii) $P(X \geq 5) = \frac{11}{20}$, iii) $E[X] = 5$, $Var[X] = 9\frac{1}{5}$.

4)

x	2	3	4	5	6
$P(X = x)$	$\frac{1}{18}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{5}{18}$	$\frac{1}{6}$

Mean = $4\frac{1}{3}$.

5) ii) $E[R] = 1\frac{5}{6}$, $Var[R] = 4\frac{5}{36}$.

6) ii) $E[X] = 4\frac{2}{3}$, $Var[X] = 8\frac{2}{9}$.

7) i) $E[S_1] = 2{\cdot}3$, $Var[S_1] = 0{\cdot}41$, ii)

$s_1 + s_2$	2	3	4
Probability	0·51	0·45	0·04

$E[S_1 + S_2] = 2{\cdot}53$.