ABSOLUTE and RELATIVE ERRORS

1) The exact roots of a certain quadratic equation are $\dfrac{3 + \sqrt{8}}{2}$ and $\dfrac{3 - \sqrt{8}}{2}$.

For each root, find, correct to 1 significant figure, the magnitude of the relative error that arises if the value used for $\sqrt{8}$ is rounded to 1 decimal place.

2) The time of swing, T seconds, of a pendulum of length l metres is given by the formula

$T = 2\pi\sqrt{\left(\dfrac{l}{g}\right)}$, where g, measured in appropriate units, is the acceleration due to gravity.

i) For a certain pendulum $l = 0 \cdot 27$ and the value of g is $9 \cdot 81$, both values being correct to 2 decimal places. Find the greatest possible value of T, giving your answer correct to 2 decimal places.

ii) Taking the values $0 \cdot 27$ and $9 \cdot 81$ as exact, calculate the relative error in T when the approximation $g = 10$ is used in the calculation of T. Give your answer as a percentage correct to 2 significant figures.

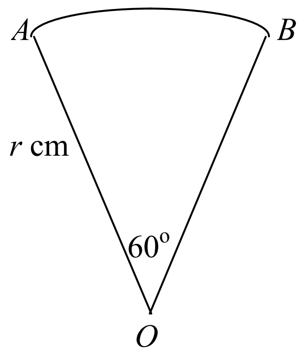
3) In the triangle ABC, $AC = 24$ mm, $BC = 27$ mm and angle $C = 69^{\circ}$.

i) Calculate the length of AB, giving your answer in millimetres correct to two decimal places.

ii) It is given that angle C is correct to the nearest degree. Taking the lengths of AC and BC as exact, calculate the greatest possible length of AB, giving your answer in millimetres correct to two decimal places.

Using this value as the true length of AB, calculate the relative error in the answer to i).

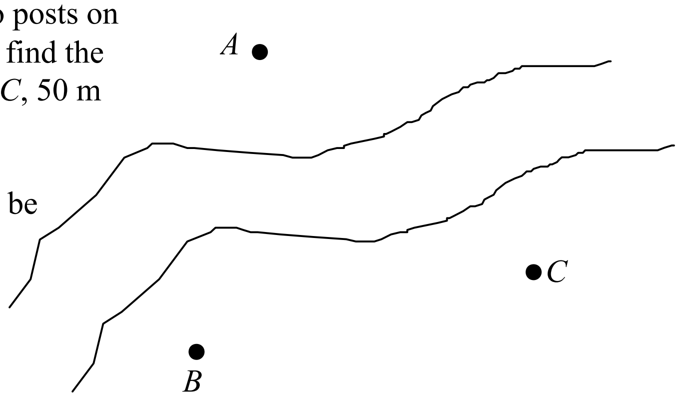
4) In the diagram, AOB is a sector of a circle with centre O and radius r cm. Arc AB subtends an angle of 60° at O.



i) Find, in terms of r and π, an expression for the area of the sector AOB.

ii) Calculate the relative error in the area of the sector AOB if angle AOB is taken to be 1 radian.

5) The diagram shows the positions A and B of two posts on opposite sides of a river. A surveyor, wishing to find the distance between A and B, places a third post at C, 50 m from B.



Angle ABC and ACB are measured and found to be 54° and 62° respectively, both being correct to the nearest degree.
Taking the value 50 m to be exact, calculate the least possible length of AB, giving your answer correct to the nearest $0 \cdot 1$ m.

ANSWERS.

1) Relative errors are 0.005 and 0.2 rounded to 1 significant figures.

2) i) 1.05 to 2 decimal places.
ii) 0.95% to 2 significant figures.

3) i) 28.99 mm.
ii) 0.0061707.

4) i) $\frac{1}{6}\pi r^2$.
ii) 0.04507.

5) 48.5 m to 1 decimal place.