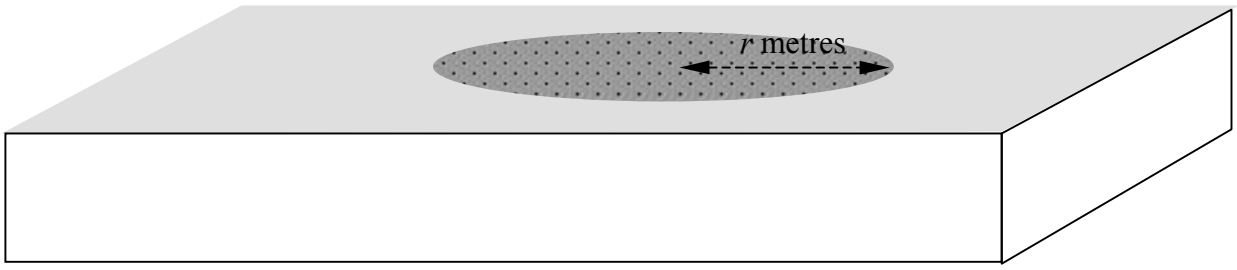
A WORKED EXAMPLE WITH DIFFERENTIAL EQUATIONS

A circular patch of oil on the surface of a pool of water has radius r metres at time t hours after the spillage occurs.



At time 2.00 p.m., 1 hour after the spillage, the radius of the patch of oil is 5 metres and it is desired to predict when it will be 12 metres in radius.

i) In a simple model, the rate of increase of r is assumed to be constant. For this model express r in terms of t and hence determine when the radius will be 12 metres.

ii) In a more refined model, the rate of increase of r is taken to be proportional to $\frac{1}{r}$. Set up a differential equation for r, involving a constant of proportionality k.

Solve the differential equation and hence show that the radius of the patch of oil is proportional to the square root of the time elapsed since the spillage.

Determine the time, to the nearest minute, at which the second model predicts that the radius of the patch of oil will be 12 metres.

ANSWERS.

i) $\frac{dr}{dt} = k$ leads, eventually, to $r = 5t$. With this model, the required time is 3.24 p.m.

ii) $\frac{dr}{dt} = \frac{k}{r}$ leads, eventually, to $r^2 = 25t$ which means that r is proportional to $\sqrt{t}$ as required.

With this model, the required time is 6.46 p.m.