CONFIDENCE INTERVALS for population means μ

1) Paint is dispensed into 1–litre cans. Let the random variable X be the volume, in litres, of paint in a can. A random sample of 100 observations $x_1, x_2, \ldots\ldots, x_{100}$ of X was made and the results may be summarised by $\sum x = 104{\cdot}0$ and $\sum x^2 = 110{\cdot}06$.

i) Calculate unbiased estimates of the mean and variance of X, giving your answers correct to an appropriate degree of accuracy.

ii) Calculate a 95% symmetric confidence interval for the mean volume of paint in a can, giving your answer correct to an appropriate degree of accuracy.

2) The amount of nicotine, in milligrams, in a cigarette of a certain brand is normally distributed with mean μ and standard deviation 2·5. A random sample of 10 cigarettes yielded a mean nicotine value of 18·4. Obtain a symmetric 90% confidence interval for μ, giving values to 3 significant figures.

Give a reason why the value of μ might not be inside this interval.

3) The mass of a certain brand of chocolate bar has a normal distribution with mean μ grams and standard deviation 0·85 grams. The masses, in grams, of 5 randomly chosen bars are

124·31, 125·14, 124·23, 125·41, 125·76.

Calculate a symmetric 90% confidence interval for μ, giving the end–points correct to 2 decimal places.

Forty random samples of 5 bars are taken, and a 90% confidence interval for μ is calculated for each sample. Find the expected number of intervals that do not contain μ.

4) On a certain day, depth soundings were taken at 80 points chosen at random on a lake. The depths, x metres, at these 80 points are summarised by $\sum x = 993{\cdot}6$, $\sum x^2 = 14121{\cdot}0$.

i) Calculate an unbiased estimate for the variance of the depth of the lake, giving the answer correct to 2 decimal places.

ii) Obtain a symmetric 90% confidence interval for the mean depth, μ metres, of the lake, giving the end–points of the interval correct to 2 decimal places.

iii) Estimate the number of depth soundings that would have to be made in order that a symmetric 90% confidence interval for μ should have a width of 1·5. You may assume that the variance estimate obtained in part i) is reliable.

iv) During the night following the taking of the measurements there was a storm and it was reported that the level of the lake rose by 2·4 cm. In the light of this information, revise your answers to parts i) and ii) as necessary.

5) The patients at the Eye Clinic of a certain hospital thought that the waiting time between arriving at the clinic and being seen by the medical staff was excessive. After many complaints the management measured the waiting times, t minutes, for a random sample of 70 patients, with the following results.

$$\sum t = 1911, \quad \sum t^2 = 79\,006.$$

i) Calculate an unbiased estimate for the variance of the waiting time of all patients attending the clinic, giving your answer correct to 3 significant figures.

ii) Find a symmetric 95% confidence interval for the mean waiting time of all patients attending the clinic.

6) Each of a random sample of 50 one–pound coins was weighed and their masses, x grams, are summarised by

$$\sum x = 474{\cdot}51, \qquad \sum x^2 = 4503{\cdot}8276.$$

i) Use an unbiased estimate of variance to calculate an approximate 90% confidence interval for the mean mass (in grams) of all one–pound coins, giving the end–values of the interval to 2 decimal places.

ii) Estimate the size of a random sample of one–pound coins that would be required to give a 95% confidence interval whose width is half that of the interval calculated in i).

iii) It was found later that the scales were consistently underweighing by 0·05 grams. State which of the results of i) and ii) should be amended and which should not. Give the amended values.

ANSWERS.

1) i) $\hat{\mu} = 1.04$, $\hat{\sigma}^2 = 0.0191919\ldots$
ii) $1.013 < \mu < 1.067$.

2) $17.1 < \mu < 19.7$.
μ might not be in this interval because it is only a 90% confidence interval. It might be one of the 10% of such confidence intervals which do not contain μ.

3) $124.34 < \mu < 125.60$.
4 such intervals are not expected to contain μ.

4) i) $\hat{\sigma}^2 = 22.537823$.
ii) $11.55 < \mu < 13.29$.
iii) At least 109 soundings would need to be taken.
iv) $\hat{\sigma}^2$ does not change. The 90% confidence interval would need to be increased by 2.4 cm which equals 0.024 m.
New confidence interval is given by $11.57 < \mu < 13.32$.

5) i) $\hat{\sigma}^2 = 389$ to 3 significant figures.
ii) $22.68 < \mu < 31.92$.

6) i) $\hat{\sigma}^2 = 0.0129142$. $9.46 < \mu < 9.52$.
ii) Approximately 221.
iii) $\hat{\sigma}^2$ does not change. Confidence interval will increase by 0.05 to $9.51 < \mu < 9.57$.
Sample size of 221 would not change.