

BINOMIAL DISTRIBUTIONS

1. If $X \sim \text{Bin}(6, 0.3)$, find
 - a) $P(X = 1)$
 - b) $P(X = 5)$
 - c) $P(X \text{ is an odd number})$

2. The probability that a marksman scores a bull's eye when he shoots at a target is 0.55 .
Find the probability that in 8 attempts,
 - a) he misses the bull 5 times.
 - b) he misses the bull at least 5 times.
{Think carefully about this one, it's trying to catch you out!}

3. A large box contains many plastic syringes, but previous experience indicates that 10% of the syringes in the box are defective. 5 syringes are taken at random from the box. Use a binomial model to calculate, giving your answers correct to three decimal places, the probability that
 - i) none of the 5 syringes is defective,
 - ii) at least 2 syringes out of the 5 are defective.

Discuss the validity of the binomial model in this context.

4. When I try to send a fax, the probability that I can successfully send it is 0.85 . I try to send 8 faxes. Use a binomial model to find the probability that I can successfully send at least 7 of the faxes.

5. In a certain town, 18% of all twenty-year-olds are in further education. A selection of 10 twenty-year-olds from the town is made at random. Use a binomial model to calculate the probability that two or more are in further education.

6. It is given that $X \sim \text{Bin}(8, 0.3)$.
 - i) Find $P(X > 0)$.
 - ii) State the value of $E[X]$.

7. A do-it-yourself winerack kit contains 33 rods, which have to be fitted into appropriate sockets. The number of badly fitting rods in a randomly chosen kit is denoted by X . State what needs to be assumed about the rods in the kit in order to model the distribution of X by a binomial distribution.

Given that the probability of any rod fitting badly is 0.05 , use a binomial distribution

- i) to find $P(X \geq 3)$
- ii) to obtain the mean and variance of X , giving your answers to 3 significant figures.

8. A university admissions tutor knows that, each year, on average only 40% of applicants offered places on a particular course eventually take up their places.

One year the tutor makes offers to seven applicants for this course, and wishes to calculate the probability that more than three of the seven applicants eventually take up the places offered.

Name a probability distribution that could be used for modelling this situation, and state one assumption necessary for this model to be valid.

Use the model to calculate the required probability.

9. It is given that $X \sim \text{Bin}(n, p)$.

- i) Write down $E[X]$ and $\text{Var}[X]$ in terms of n and p .
- ii) Given that $E[X] = 10$ and $\text{Var}[X] = 8$, find the value of p .
{**Hint** look at the expression $\frac{\text{Var}[X]}{E[X]}$ in terms of n and p .}

ANSWERS.

1. a) 0.302526. b) 0.010206. c) 0.497952.
2. a) 0.1719248. b) 0.2603807.
3. i) 0.59049. ii) 0.08146.

Binomial may not be entirely appropriate since all the syringes might be from the same batch, that is if one is defective, ALL might thus be defective. Hence *independence* might be a problem.

4. 0.657183.

5. 0.5608367.

6. i) 0.9423519. ii) 2.4.

7. 'Badly fitting' rods must be independent of one another.

i) 0.2271931. ii) $E[X] = 1.65$, $\text{Var}[X] = 1.57$.

8. Bin(7, 0.4) providing applicants choices are independent of one another.

0.2897920.

9. i) $E[X] = np$, $\text{Var}[X] = np(1 - p)$.
ii) $p = 0.2$.