DETERMINANTS and INVERSES of 2x2 MATRICES

1. Evaluate the following determinants.

a) $\begin{vmatrix} 12 & 5 \\ 27 & 11 \end{vmatrix}$ b) $\begin{vmatrix} 36 & -9 \\ -4 & 1 \end{vmatrix}$ c) $\begin{vmatrix} x+2 & 4-x \\ x & 7 \end{vmatrix}$.

2. Find the inverses of A, B and C, where

$$A = \begin{pmatrix} 7 & 19 \\ 2 & 6 \end{pmatrix}, \quad B = \begin{pmatrix} 5 & \frac{1}{2} \\ 20 & 2 \end{pmatrix}, \quad C = \begin{pmatrix} a & -4 \\ 1 & 3 \end{pmatrix}.$$

3. Find all values of k for which the matrix $M = \begin{pmatrix} 3 & k+2 \\ -k & k-2 \end{pmatrix}$ is singular (i.e. non-invertible).

4. By finding $\begin{pmatrix} 3 & 7 \\ 1 & 6 \end{pmatrix}^{-1}$, solve the equations

$$3x + 7y = 9$$
$$x + 6y = 5.$$

5. Given $X = \begin{pmatrix} 2 & 3 \\ -2 & 1 \end{pmatrix}$, show that $X^2 - 3X + 8I = \mathbf{\underline{O}}$, where I is the 2x2 identity matrix and $\mathbf{\underline{O}}$ is the zero matrix.

Deduce that $X^{-1} = \frac{1}{8}(3I - X)$. {**Hint:** Multiply the above equation by X^{-1}.}

6. Given $A = \begin{pmatrix} 3 & 7 \\ 1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & -1 \\ -3 & 1 \end{pmatrix}$, find AB and $(AB)^{-1}$.

a) Verify that, in this case, $(AB)^{-1} \neq A^{-1}B^{-1}$.

b) Verify that, in this case, $(AB)^{-1} = B^{-1}A^{-1}$.

7. Show that all matrices of the form $\begin{pmatrix} 6a+b & a \\ 3a & b \end{pmatrix}$ commute with $A = \begin{pmatrix} 2 & 1 \\ 3 & -4 \end{pmatrix}$.

{A and B commute if $AB = BA$.}

ANSWERS.

1. a) -3 b) 0 c) $x^2 + 3x + 14$.

2. $A^{-1} = \frac{1}{4}\begin{pmatrix} 6 & -19 \\ -2 & 7 \end{pmatrix}$, B^{-1} does not exist, $C^{-1} = \frac{1}{3a+4}\begin{pmatrix} 3 & 4 \\ -1 & a \end{pmatrix}$.

3. $k = -6$ or 1.

4. $x = \frac{19}{11}$, $y = \frac{6}{11}$.