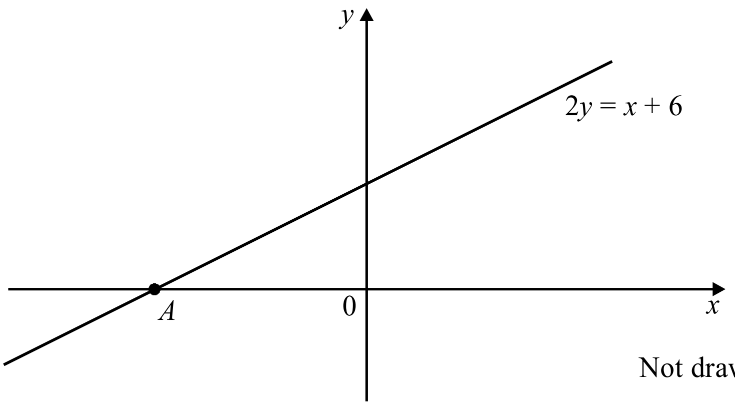
GCSE MATHEMATICS Higher Tier, topic sheet.
STRAIGHT LINES (excludes perpendicular lines)

$y = mx + c$ represents the straight line with **gradient** m and **intercept** c.

1. The diagram shows the line $2y = x + 6$.



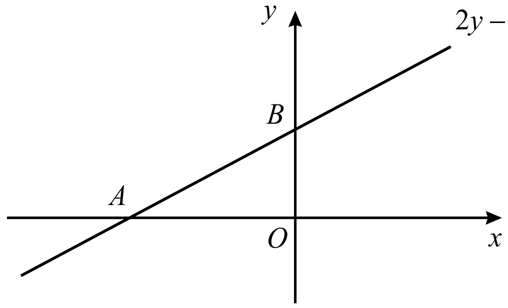
(a) What are the coordinates of the point marked A?

(b) What is the gradient of the line $2y = x + 6$?

(c) On the diagram draw another line that has the same gradient as $2y = x + 6$.

(d) The line $y = -x$ crosses the line $2y = x + 6$ at the point B.
Calculate the coordinates of B.

2. A sketch of the line $2y - x = 4$ is shown.
The line crosses the axes at A and B.



(a) Calculate the coordinates of A and B.

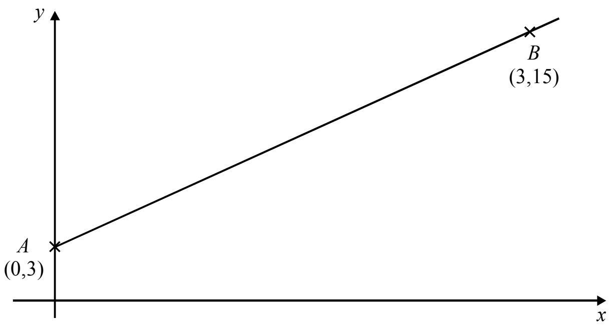
(b) Calculate the gradient of the line AB.

3. A is the point (0,2) and B is the point (3,5).

(a) Find the **exact** length of AB.

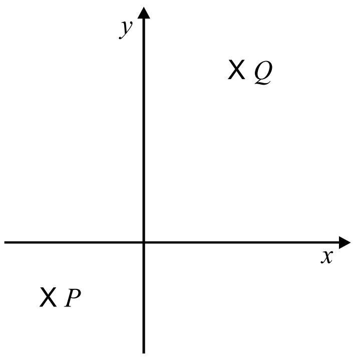
(b) Find the equation of the line joining the points A and B.

4. The diagram shows the points $A(0,3)$ and $B(3,15)$.



Find the equation of the line AB.

5. The sketch below shows the points $P(-3, -2)$ and $Q(5, 13)$.



not drawn accurately

(a) Calculate the length of PQ.

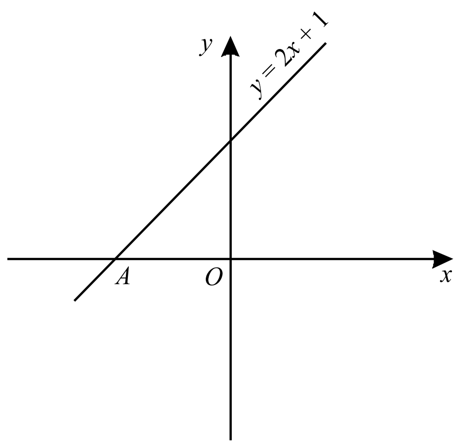
(b) Find the equation of the line which is parallel to PQ and passes through the point (0, 2)

6. The sketch shows the graph of $y = 2x + 1$.

(i) Give the co-ordinates of the point A.

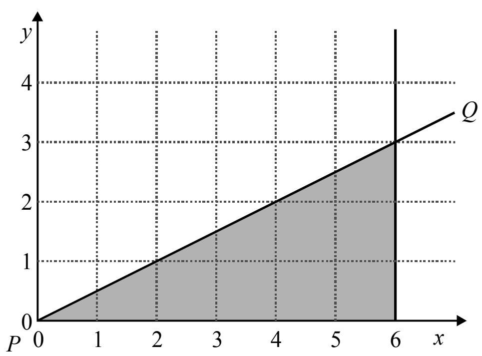
(ii) On the grid above, sketch the graph of $y = 2x - 1$.

(iii) From your graphs explain why the equation $y = 2x + 1$ and $y = 2x - 1$ cannot be solved simultaneously.



7. (a) Find an equation for the line PQ in the diagram shown.

(b) Write down **three** inequalities which together describe the shaded area.



SOLUTIONS / ANSWERS.

1. (a) Put $y = 0$ to get $0 = x + 6$ and thus $x = -6$.
Therefore $A = (-6, 0)$.

(b) We have $2y = x + 6$ which means that $y = \frac{1}{2}x + 3$.
Therefore the gradient = $\frac{1}{2}$.

(c) Draw any line parallel to the original line.

(d) Solve $2y = x + 6$
and $y = -x$ simultaneously.

We have $2(-x) = x + 6 \Rightarrow -2x = x + 6$
$-3x = 6$
and thus $x = -2$ and $y = -x = 2$.
$B = (-2, 2)$.

2. (a) <u>Put $x = 0$ to find B:</u> $2y - 0 = 4 \Rightarrow y = 2$. Hence $B = (0, 2)$.
<u>Put $y = 0$ to find A:</u> $2 \times 0 - x = 4 \Rightarrow x = -4$. Hence $A = (-4, 0)$.

(b) Gradient of $AB = \dfrac{\textit{difference in } y-\textit{coordinates}}{\textit{difference in } x-\textit{coordinates}} = \dfrac{2}{4} = \dfrac{1}{2}$.

3. (a) Draw a diagram!
Using Pythagoras we have $AB^2 = 3^2 + 3^2 = 9 + 9 = 18$.
Thus $AB = \sqrt{18}$.

(b) Gradient of $AB = \dfrac{\textit{difference in } y-\textit{coordinates}}{\textit{difference in } x-\textit{coordinates}} = \dfrac{3}{3} = 1$.
Therefore the equation is given by: $y = mx + c$ (m = gradient = 1)
$y = x + c$.
Since line passes through (0, 2), put $x = 0$, $y = 2$ to get $2 = 0 + c$ and thus $c = 2$.
Answer: $y = x + 2$.

4. Gradient of $AB = \dfrac{\textit{difference in } y-\textit{coordinates}}{\textit{difference in } x-\textit{coordinates}} = \dfrac{12}{3} = 4$.
Therefore the equation is given by: $y = mx + c$ (m = gradient = 4)
$y = 4x + c$.
Since line passes through (0, 3), put $x = 0$, $y = 3$ to get $3 = 4 \times 0 + c$ and thus $c = 3$.
Answer: $y = 4x + 3$.

5. (a) Using Pythagoras we have $PQ^2 = 8^2 + 15^2 = 64 + 225 = 289$.
Thus $PQ = \sqrt{289} = 17$.

(b) Gradient of $PQ = \dfrac{\textit{difference in } y-\textit{coordinates}}{\textit{difference in } x-\textit{coordinates}} = \dfrac{15}{8}$.
Therefore the equation is given by: $y = mx + c$ (m = gradient = $\frac{15}{8}$)
$y = \frac{15}{8}x + c$.
Since line passes through (0, 2), we must have that c = intercept = 2.
Answer: $y = \frac{15}{8}x + 2$.

6. (i) Put $y = 0$ to get $0 = 2x + 1$ and thus $x = -\frac{1}{2}$.

Therefore $A = (-\frac{1}{2}, 0)$.

(ii) Sketch the graph of $y = 2x - 1$ which passes through $(0, -1)$ and $(1, 1)$ for example.

(iii) Since the graphs are parallel, and therefore never intersect, the simultaneous equations will thus have no solutions.

7. (a) Gradient of $PQ = \dfrac{difference\ in\ y-coordinates}{difference\ in\ x-coordinates} = \dfrac{3}{6} = \dfrac{1}{2}$.

Therefore the equation is given by: $y = mx + c$ (m = gradient $= \frac{1}{2}$)

$y = \frac{1}{2}x + c$.

Since line passes through $(0, 0)$, we must have that c = intercept = 0.

Answer: $y = \frac{1}{2}x$.

(b) $y \geq 0,\ x \leq 6,\ y \leq \frac{1}{2}x$.