INDICES and STANDARD FORM

As mentioned already in the chapter on Factors and Prime numbers, the word '*index*' is the proper mathematical term for *power*. So, nothing to fear here.

As for standard form; this is a rather strange topic, but one which most students find relatively straight forward because there is so little to remember.

The key to standard form can be wrenched from the following calculations:

$$6.5 \times 10^3 = 6.5 \times 10 \times 10 \times 10 \quad \text{which equals} \quad 6500$$
$$6.5 \times 10^2 = 6.5 \times 10 \times 10 \quad \text{which equals} \quad 650$$
$$6.5 \times 10^1 = 6.5 \times 10 \quad \text{which equals} \quad 65.$$

(Recall that multiplying by 10 simply *shifts* the digits one place to the left etc.)

Continuing this pattern would suggest the following results

$$6.5 \times 10^0 = 6.5$$

and $$6.5 \times 10^{-1} = 0.65$$

etc.

So, we see that, for example, 6.5×10^{-1} is the same as $6.5 \div 10^1$ etc.

Standard form is essentially the reverse of the above in that we take a number such as 350 and write it in the form $3.5 \times 10 \times 10$ or, more succinctly, 3.5×10^2.

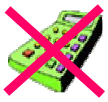
A number is said to be in *standard form* if it is written in the form

$$(\quad\quad) \times 10^{(\quad)}$$

where the value in the first bracket must be between 1 and 10.

E.g. the number 3.8×10^3 is said to be in standard form, whilst 38×10^2 is not in standard form since 38 is not between 1 and 10.

Example 1.
Write the following numbers in standard form.



a) 560, b) 56, c) 5.6, d) 0.56.

Solution.

a) In searching for a number between 1 and 10 to represent 560 we inevitably arrive at 5.60.
It is then a short step to get to the fact that $560 = 5.6 \times 100$ or 5.6×10^2.

b) In a similar way to a), we see that $56 = 5.6 \times 10^1$,

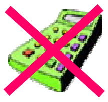
The answers for both c) and d) are obtained by following the obvious pattern in a) and b).

c) $5.6 = 5.6 \times 10^0$.
d) $0.56 = 5.6 \times 10^{-1}$.

For a lot of people, standard form simply becomes a process of searching for a number between 1 and 10 and then counting how many times the digits must move in order to return to the original number etc. Well it works, and this close to an exam, I'm not going to argue!

Try the following example for practice.

Example 2.
Write the following numbers in standard form.



a) 3460, b) 105, c) 4.21, d) 0.016.

Answers.

a) $3460 = 3.46 \times 10^3$ b) $105 = 1.05 \times 10^2$

c) $4.21 = 4.21 \times 10^0$ d) $0.016 = 1.6 \times 10^{-2}$.

NOTE.
Computers and calculators use standard form when storing numbers in their memories since it allows for the storage of very large and very small decimal numbers.

For example, the number 18 000 000 000 000 when written in standard form equates to $\mathbf{1.8} \times 10^{\mathbf{13}}$. A computer would simply store the values **1.8** and **13** as a way of remembering this value.

Easy enough ?

To make things easier, let us employ a calculator to check our answers.

All we need are the *power* button, usually labelled [x^y] , or sometimes [^] , and the *plus / minus* button (not the subtract key).

For example, to key in 1.05×10^2 on a *newish* Casio calculator, press the following buttons:

[1] [.] [0] [5] [×] [1] [0] [x^y] [2] [=]

ANSWER should equal 105.

Now try 1.6×10^{-2} on your calculator.

Press [1] [.] [6] [×] [1] [0] [x^y] [–] [2] [=]

ANSWER should equal 0.016.

The *minus* key. NOT the subtract key.

Now we can check our answers.

WARNING. If your calculator display shows you a number like 1.6^{-02}, then it is being *lazy*! Your calculator is actually referring to the number given by 1.6×10^{-2} (which is the same as 0.016), but thinks it is being clever by leaving out the ×10 ... etc.

Similarly 2.4^{-03} on your calculator display refers to the number 2.4×10^{-3} (or 0.0024).

So, look carefully at your calculator display !

Example 3.

Work out the following. **Give your answers in standard form.**

a) $(3 \times 10^7) + (1.4 \times 10^5)$, b) $(9.9 \times 10^6) \times (2 \times 10^{-3})$,

c) $\dfrac{4.5 \times 10^{-7}}{3 \times 10^{-3}}$.

Solution.

a) On a calculator (assuming it works the same way as a *newish* Casio!), press the following keys:

$(\; 3 \times 1 \; 0 \; x^y \; 7) + (\; 1 \; . \; 4 \times 1 \; 0 \; x^y \; 5 \;) =$

to get an answer of 30140000. Now convert to standard form as requested to get 3.014×10^7.

b) Again simply use a calculator, including the bracket buttons, to get an answer of 19800.

Convert to standard form to get 1.98×10^4.

c) Comment. This question is trying to catch us out by not putting brackets in. Let us rewrite it correctly: $\dfrac{(4.5 \times 10^{-7})}{(3 \times 10^{-3})}$. Now we simply grab a calculator!

Press the following keys:

$(\; 4 \; . \; 5 \times 1 \; 0 \; x^y \; - 7 \;) \div (\; 3 \times 1 \; 0 \; x^y \; - 3 \;) =$

Minus button! NOT the subtract key.

Answer = 0.00015 or, in standard form, 1.5×10^{-4}.

Comment.

If your calculator is being *lazy* and displays the answer, 1.5^{-04}, then simply remember to put in the ×10 and give your answer as 1.5×10^{-4}. You see, not so daft after all these calculators !

These days, GCSE examiners have a rather nasty habit of occasionally throwing the above types of calculations in the direction of the NON-CALCULATOR papers!

Still, not to worry. I've included a whole bunch of these in the exercise below, which I advise you to stare at for a few days and nights and seek out your teacher if you require a more vocal explanation.

Example 4.
Light travels at approximately 298 000 km per second.

a) Find the speed of light in km per hour and write your answer in **standard form**.

b) The distance between the Sun and The Earth is approximately 143 000 000 km. Find, to the nearest minute, the time taken for sunlight to reach the Earth.

Solution.

a) We are given that light travels 298 000 km every second. Well this means that light must travel $60 \times 298000 = 17880000$ km every minute.
Therefore, light must travel 60×17880000 km = 1072800000 km every hour.

Converting to standard form gives us an answer of 1.0728×10^9 km per hour.

{Again, if your calculator simply shows you an answer of 1.0728^{09} then don't forget to write in the ×10 ... etc.}

b) We know from part a) that light travels 17880000 km every minute.
Thus, the number of minutes required for light to travel the 143 000 000 km between the Sun and the Earth is given by $\frac{143000000}{17880000} = 7.9977...$ minutes.

This gives a time of 8 minutes to the nearest minute.

Exercise 1.

1) Write the following numbers in standard form.

a)	35,	b)	4210,	c)	125·002,
d)	9·2,	e)	125000,	f)	8,
g)	0·025,	h)	0·00167,	i)	100,
j)	1,	k)	251.		

2) Work out the following. **Give your answers in standard form.**

a) $(3.5 \times 10^{7}) + (2{\cdot}6 \times 10^{5})$, b) $(3{\cdot}9 \times 10^{6}) \times (2.1 \times 10^{-3})$,

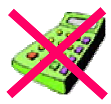
c) $\dfrac{(3\times10^{2})}{(3\times10^{-3})}$, d) $(4{\cdot}25\times10^{6}) - (3{\cdot}456\times10^{4})$,

e) $(7{\cdot}29\times10^{12}) \div (3{\cdot}5\times10^{8})$, f) $\dfrac{(36\times10^{4})}{(8\cdot5\times10^{3})}$,

g) $(3.2 \times 10^{4}) \times (2.5 \times 10^{3})$, h) $\dfrac{8.6 \times 10^{6}}{4.3 \times 10^{-3}}$,

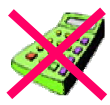
i) $(3.6 \times 10^{5})^{3}$.

3) The following numbers have been written in standard form. Write **each** in decimal form. a) 3.5×10^{6}, b) 8.2×10^{-3}.



4) Find, in standard form, the values of

a) $(3 \times 10^{2}) \times (2 \times 10^{3})$, b) $(4 \times 10^{2}) \times (2 \times 10^{4})$,
c) $(3 \times 10^{3}) \times (3 \times 10^{5})$, d) $(3.1 \times 10^{7}) \times (2 \times 10^{3})$,
e) $(4.3 \times 10^{2}) \times (2 \times 10^{6})$, f) $(3.2 \times 10^{5}) \times (3 \times 10^{-2})$,
g) $(3.3 \times 10^{-2}) \times (3 \times 10^{-1})$, h) $(4.8 \times 10^{-3}) \times (2 \times 10^{-2})$,
i) $(1.5 \times 10^{-2}) \times (4 \times 10^{5})$.



5) Find, in standard form, the values of

a) $(6 \times 10^{2}) \times (2 \times 10^{3})$, b) $(9 \times 10^{2}) \times (2 \times 10^{4})$,
c) $(6 \times 10^{3}) \times (3 \times 10^{5})$, d) $(8 \times 10^{7}) \times (5 \times 10^{3})$,
e) $(25 \times 10^{2}) \times (5 \times 10^{6})$, f) $(30 \times 10^{5}) \times (4 \times 10^{-2})$,
g) $(3.1 \times 10^{4}) \times (6 \times 10^{-1})$, h) $(7.8 \times 10^{5}) \times (2 \times 10^{-2})$,
i) $(12 \times 10^{3}) \times (11 \times 10^{5})$.

ANSWERS / SOLUTIONS.

1)	a)	$3{\cdot}5 \times 10^1$,	b)	$4{\cdot}21 \times 10^3$,	c)	$1{\cdot}25002 \times 10^2$,
	d)	$9{\cdot}2 \times 10^0$,	e)	$1{\cdot}25 \times 10^5$,	f)	$8{\times}10^0$,
	g)	$2{\cdot}5 \times 10^{-2}$,	h)	$1{\cdot}67 \times 10^{-3}$,	i)	1×10^2,
	j)	1×10^0,	k) $2{\cdot}51 \times 10^2$.			

NOTE that there appears to be 2 possible answers for the number 100, namely 1×10^2 or 10×10^1.
Only 1×10^2 is correct since we do not allow a $10 \times$ .... etc.

2)	a)	3.526×10^7,	b)	8.19×10^3,	c)	1×10^5,
	d)	$4{\cdot}21544 \times 10^6$,	e)	$2{\cdot}0828571 \times 10^4$,	f)	$4{\cdot}2352941 \times 10^1$,
	g)	8×10^7,	h)	2×10^9,	i)	4.6656×10^{16}.

Remember to put the brackets in part h). Also, beware of your calculator leaving out the $\times 10$ ... etc.

3)	a)	3500000,	b)	0.0082.

Comment regarding questions 4 and 5). **NO CALCULATORS !!!**
These do not turn up frequently enough that you should be overly concerned if you find them difficult. Just be prepared to have a go.

For example, $(3 \times 10^2) \times (2 \times 10^3)$ can be written $(3 \times 10 \times 10) \times (2 \times 10 \times 10 \times 10)$ which simplifies to give the answer of 6×10^5.

$(3 \times 10^2) \times (2 \times 10^3)$

Simply staring at the questions in conjunction with the answers below might do the job! Failing that, *ask your teacher* !

4)	a)	6×10^5,	b)	8×10^6,	c)	9×10^8,
	d)	6.2×10^{10},	e)	8.6×10^8,	f)	9.6×10^3,
	g)	9.9×10^{-3},	h)	9.6×10^{-5},	i)	6×10^3.

5)	a)	1.2×10^6,	b)	1.8×10^7,	c)	1.8×10^9,
	d)	4×10^{11},	e)	1.25×10^{10},	f)	1.20×10^5,
	g)	1.86×10^4,	h)	1.56×10^4,	i) 1.32×10^{10}.	